Spring 2005

Midterm - solution key

(I) individual states: k_t

a) aggregate states: K_t, T_t

control variables: k_{t+1}, h_t

b) $w_t h_t + r_t k_t = c_t + k_{t+1} - (1-\delta) k_t$

Households have to take their investment decisions at date t (for date $t+1$). If the capital is to be destroyed, that would be the case at date $t+1$.

c) Firms choose k_t, h_t to $\max_{h_t, k_t} f(h_t, k_t) - w_t h_t - r_t k_t$

FOC $[h_t]$ $\quad f_1(h_t, k_t) = w_t$

FOC $[k_t]$ $\quad f_2(h_t, k_t) = r_t$

1) Clearly, labour is inelastically supplied

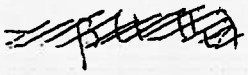
$$V(k_t, K_t, T_t) = \max_{k_{t+1}} \left\{ u(c_t) + \beta \left[\pi_t V(k_t, K_t, 0) + (1-\pi_t) V(k_{t+1}, K_{t+1}, T_{t+1}) \right] \right\}$$

(where $\pi_t = \frac{\pi}{1+T_t}$)

FOC $[k_{t+1}]$ $\quad u'(c_t) = \beta(1-\pi_t) V_1(k_{t+1}, K_{t+1}, T_{t+1})$

ET $\quad V_1(k_t, K_t, T_t) = u'(c_t) \left[r_t + 1 - \delta \right]$

$$\Rightarrow \quad u'(c_t) = \beta(1-\pi_t)\, u'(c_{t+1}) \left[r_{t+1} + 1 - \delta \right]$$



f) the FOC is

$$u'(c_t) = \beta(1-\pi_t)\, u'(c_{t+1}) \left[r_{t+1} + 1 - \delta \right]$$

└ expected return from delaying consumption

As $\pi_t \uparrow$, $\pi_t \downarrow$, return from delaying consumption $\Rightarrow$ investment goes up