

Prépare ~~le~~ ~~test~~  
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**A** Question 1:

Provide the conditions for an equilibrium to exist in the Pissarides model (exogenous separations). You may have to make some assumptions about the matching function to get the results (hint: get some inspiration from a Cobb-Douglas matching function).

**B** Question 2:

How would you write the value function in the Mortensen-Pissarides model (endogenous separations), if you allowed a third state for the worker: "out of the labor force" (see one of the footnotes in chapter 3)? Assume that workers can leave the labor market and "home produce". Home production is stochastic as well. According to a Poisson process with rate  $\mu$ , a new shock to home production may arrive, in which case, the new value is drawn from a distribution  $G$ . When the shock to home production is  $\xi$ , the value of home output is  $\xi$ . To fix ideas, assume that within a period  $dt$ : (1) there may or may not be a realization of the Poisson process, governing shocks to market production ( $\lambda$ ), (2) then, there may or may not be a realization of the Poisson process governing shocks to home production ( $\mu$ ). Also, within a period  $dt$ , there may or may not be a realization of the Poisson process governing the matching technology. Assume that you cannot go directly from "out of the labor force" to "employment". You need to go through "unemployment". For simplicity, you can write directly the value function in steady state.

**C** Question 3:

In the Mortensen-Pissarides model, what happens if  $\beta \rightarrow 0$  or if  $\beta \rightarrow 1$ ? Provide some intuition.

**D Question:**

In this question, we are modelling a labor market with frictions, where wages are determined by a union. The timing is the following. In each period, the union sets a fixed wage  $w_u$ , that the firm has to pay each worker it hires. The union anticipates how the wage it sets affects the firms' hiring and firing decisions, and it chooses the wage that maximizes its objective function. The latter is a weighted average of the unionized worker's value of search (with weight equal to the unemployment rate  $U\%$ ) and of his value of a match (with weight  $1 - U\%$ ).

The meeting and production technologies are standard. The number of meetings in a period is given by a CRS function  $M(U, V)$ , which depends on the number of unemployed workers and the number of vacancies. Workers are either employed or looking for a job. Similarly, jobs can either be vacant or producing. Firms have to post vacancies to find workers. Let  $\theta = \frac{V}{U}$  be the market tightness. Because of random matching, the meeting probabilities only depend on market tightness.

The productivity of job/worker pairs is idiosyncratic. Denote match output by  $x$ . Every period, a new shock hits the match with probability  $\lambda$ . Conditional on a new shock,  $x$  is drawn from a distribution  $F(z)$  ( $z \in [\underline{x}, \bar{x}]$ ). Assume that newly created jobs start at the highest productivity  $\bar{x}$ .

Some final notation before starting:  $S^f$  (value of a vacancy to a firm),  $M^f(x)$  (value of a match to a firm, as a function of  $x$ ),  $S^w$  (value of search to a worker),  $M^w(x)$  (value of employment to a worker, as a function of  $x$ ).  $r$  is the discount rate,  $c$  the cost of posting a vacancy (per unit of time) and  $b$  the worker's income during search.

- (a) What are the decision variables for the firms?
- (b) What are the decision variables for the workers?
- (c) Write down the equations defining the value functions  $S^f, M^f(x), S^w$  and  $M^w(x)$ , conditional on  $w_u$ . Solve for equilibrium (conditional on  $w_u$ ).
- (d) What is the union's maximization problem?

**Question 3 (30 points): A model of temporary layoffs.**

This question is based on the Mortensen & Pissarides matching model (i.e. with endogenous separations.) Since we consider a model in continuous time, everything - number of matches, output, wages, value of leisure, all costs, Poisson rate - is to be understood as measured per unit of time. For a given stock of unemployed  $U$  and a given stock of vacancies  $V$ , there is a meeting technology given by  $M(U, V)$  where  $\theta = V/U$ , with the usual properties. Once matched, the production technology is stochastic, i.e. output is given by an idiosyncratic term  $\varepsilon$  drawn from a distribution  $F(\varepsilon)$ . All matches initially start at the highest productivity value  $\varepsilon_u$ . Productive matches may be hit by new idiosyncratic shocks. This is governed by a Poisson process with a rate  $\lambda$ . Conditional on being hit by a new shock, the new productivity is drawn from  $F(\cdot)$ . Hence, every period the wage  $w(\varepsilon)$  may be re-negotiated. Bargaining is determined by the usual Nash bargaining solution with equal bargaining power to the firm and the worker. The future is discounted at rate  $r$  by both firms and workers.

**Firms:** There is a continuum of firms, whose size is determined by a free entry condition. Search is costly to firms, who have to pay a cost  $c$  while searching.

**Workers:** There is a continuum of workers of mass 1. Workers have a value of leisure  $b$  which they enjoy only when they are not matched.

We want to model "temporary layoffs," that is the possibility of laying off a worker for a short period of time with the idea of recalling that same particular worker later on if times are better. To be precise, when a worker and a

firm are in a regular match and are hit by a new shock, the firm can decide to (i) stay matched, (ii) fire the worker for good which implies a firing tax  $t$  to the firm,<sup>1</sup> or (iii) lay the worker off temporarily and not pay  $t$ . That last option implies that the firm keeps a relationship with the worker to be able to recall that very worker later on. Keeping a relationship with that very worker is costly, implying a cost  $\kappa$  to the firm.<sup>2</sup> We assume that temporary layoff means that even though the match is not active anymore, the specific worker and firm keep a relationship and their inactive match is subject to the same idiosyncratic productivity process as an active match would. For analytical tractability, we assume that the layoff can remain temporary until a new shock hits the match. At that point, firms cannot use that temporary layoff option any longer and have to either resume an active match at the new productivity or lay the worker off permanently and incur the firing tax  $t$ . Notice that temporary laid off workers received their value of leisure  $b$  and can also search for a full time job at no cost.

(3a) We are only interested in how the nature of the problem would be changed, not in solving for the new equilibrium. List all the state variables, control variables and value functions of the problem.

(3b) Write all the value functions in flow terms, that as a sum of a current utility and a capital gain/loss. Do not solve the system. If you want for simplicity, you can derive one value function, the simplest one, and write down the other ones.

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<sup>1</sup>As assumed in class, a firing tax  $t$  is not transferred to the worker.

<sup>2</sup>That cost may be understood as necessary to keep in touch with the worker. Or it could be for example that the firm does not employ that worker for a while, but keeps paying some kind of benefits to that worker, or even employ that worker with reduced hours. For the case here, we will assume the first option, which implies that the cost  $\kappa$  is not received by the worker.