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HETEROGENEOUS MATCHING WITH TRANSFERABLE UTILITY: TWO LABOR MARKET APPLICATIONS*

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A labor market model under search frictions is developed, where participants are heterogeneous in productivity and the decision of which type of agents to match with is endogenized. Two applications are studied. It is observed that countries with high (low) unemployment tend to exhibit low (high) wage dispersion. And there is evidence showing that individual and firm characteristics have more explanatory power for the French than for the American wage data. Matching patterns can account for these two observations. In the absence of a minimum wage, I thus provide a theory of endogenous wage compression.

1. MOTIVATION

The article looks at how agents, who differ with respect to their productivities, decide to match in an economy characterized by search frictions. When this is the case, a central question is: Which matching patterns are sustainable in equilibrium? In other terms, who matches with whom? The question is asked in the context of a labor market, where workers are characterized by different skill levels and firms by different technologies. Interestingly, using this framework, it is possible to replicate two empirical regularities observed in American and European labor markets. First, it is well documented in Abraham and Houseman (1995), Bertola and Ichino (1995), and Katz et al. (1995) that the former is characterized by low unemployment but high wage dispersion, and the latter by high unemployment but low wage dispersion. Bertola and Ichino even point out the fact that within-type wage dispersion (i.e., after controlling for education and experience) is also higher in the United States than in Europe. I show that different matching behaviors among heterogeneous agents can simultaneously explain these patterns of unemployment and wage dispersion. Second, again contrasting American and European labor markets, Abowd et al. (2001) find that, accounting for observable and unobservable heterogeneity, individual characteristics plus establishment effects explain about 20% more of the annual variation in annual wages for the

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French sample, as opposed to the American one. This is an observation, that can also be explained in the context of this model. Hence, the analysis of matching patterns across countries, although relatively ignored, is very relevant to the study of labor markets.

Matching models can be divided into two categories, depending on how the match payoffs are determined. The first one is comprised of models where nontransferable utility is assumed, as in Burdett and Coles (1997, 1999). In these, individuals take the characteristics of their potential partner as given, determining their utility in the match. A match will take place if and only if both individuals derive sufficient utility from it, given these fixed payoffs. This typically results in the creation of "classes," where individuals only match with partners of similar characteristics. The second category is comprised of models where transferable utility is assumed: A meeting between two agents creates a local surplus, whose division between the two partners is bargained over. Therefore, as long as there are gains from trade, the possibility to negotiate the division of the surplus ensures that a partner can always induce the other one to accept the match, while also retaining a positive surplus for herself. Hence, a match will take place if and only if the combined match surplus is positive. The labor market is the prototypical application of transferable utility, since firms and workers can negotiate wages to split output. The article focuses on the transferable utility case, with heterogeneous agents. We will see that this results in matching patterns that may be very different from the ones observed with nontransferable utility.

The few related papers addressing the issue of matching between heterogeneous agents with transferable utility are Burdett and Coles (1999), Sattinger (1995), and Shimer and Smith (2000). Burdett and Coles look at all the basic ingredients required for a general theory of partnership formation under several different settings: transferable utility, nontransferable utility, and match-specific heterogeneity. Sattinger looks more specifically at the case of transferable utility and focuses on how ex ante differences in worker quality may generate sorting externalities, as the workers' matching patterns affect the composition of the pool of unemployed workers in equilibrium and therefore other workers' decisions. Shimer and Smith define a search equilibrium in the case where there is a continuum of types and find sufficient conditions for existence of equilibrium and for the agents' matching sets to be convex. The present model uses a framework similar to Shimer and Smith, but its focus is on two applications. First, the model has the property that equilibria where agents match with a larger set of productivity types tend to result in higher wage dispersion and lower unemployment. Some empirical evidence is provided to support the notion that Europe may be exhibiting the kind of matching patterns resulting in higher unemployment and lower wage dispersion than in the United States. Simulations also indicate that the respective labor market policies in place result in matching patterns consistent with such observations. In the process, I thus provide a theory of endogenous wage compression. Second, the model can also explain the fact that there is more noise in American than in French wage regressions, after accounting for both observable and unobservable characteristics, solely by relying on different matching patterns. These two applications emphasize the importance of incorporating the matching behavior of heterogeneous participants in the labor market, when studying these markets across countries.

This model also contributes to the literature on wage inequality. Acemoglu (1999) and Albrecht and Vroman (2002) build models where the labor market is characterized by search frictions. They have ex ante heterogeneity in workers, but their setups differ from mine, since they assume that firms endogenously post vacancies. Although their models differ along several dimensions, they both find that equilibria with endogenous segmentation along worker skill lines result in both higher wage dispersion and unemployment than equilibria where high- and low-skill workers may accept the same type of jobs. Moreover, whereas that literature attempts to account for the recent trend in wage inequality in the United States, the present model is interested in explaining the differences in wage inequality and unemployment in the United States and Europe.[2]

The article is organized as follows. A matching equilibrium is defined in Section 2. The general characteristics of such an equilibrium are presented in Section 3, emphasizing the consequences of assuming transferable utility. The two applications previously mentioned are studied in Section 4. Finally, Section 5 concludes and presents possible future extensions.

2. MATCHING EQUILIBRIUM

The economy is composed of (i) a pool of searching agents looking for a partner to match with and (ii) a pool of matched agents who are producing and splitting the output of their match. Exogenous breakdowns in the production pool are the source of new entrants into the search pool. In addition, it is assumed that utility is fully transferable between agents in a match and that the wage is determined through bargaining. This simple setup is designed to closely replicate the workings of a labor market. The main characteristics are that heterogeneous agents are looking for partners to form a long-term relationship, where some output is to be produced and shared (no output can be produced by a single agent). In addition, the matches may be stochastically broken, in which case the search process has to resume.

Because of search frictions, finding a partner to engage in production with is a time-consuming process and agents get to meet each other only randomly according to a Poisson process. Consider that there are two productivity types, p_i, $i \in \{l, h\}$ and that there is a constant total number of agents of each type in the entire economy (notice that the model can be extended to allow for n productivity types in a straightforward fashion). There is no uncertainty about the type of agents met. These agents are referred to as partners. This is purely for simplicity, since the nature of the relationship between two partners is the same as between a worker and firm. There is no further search once the match is formed. Each type has a bargaining power $\theta = 1/2$ (in the Nash bargaining solution). The output from

[2] Kremer and Maskin (1996) and Krusell et al. (2000) look at the effects of a skill-biased technological change on wage dispersion. However, because they look at competitive economies, their models do not have implications on unemployment.

a match is determined by a strictly positive, increasing, and symmetric production function, which is assumed to be additive, so that all firms can be considered as just a worker–job pair. Denote by f_{ij} the output produced in a match between type p_i and type p_j. Hence,

$$\forall i, j, \quad f_{ij} = f_{ji}$$

$$\forall (i, j, k) \in \{l, h\}, \quad j > k \Rightarrow f_{ij} > f_{ik}$$

Denote by U_i the discounted lifetime expected value of search for an unmatched partner of type p_i, and by M_{ij} the discounted lifetime expected value of a match to a type p_i partner, when matched with a type p_j partner. When considering whether to match, the searching partner's decision is a combination of several factors. It depends on the income received during search. This is given by b_i for a type p_i partner, where $b_h \geq b_l$. It also depends on how frequent the matching opportunities are, and how long the matches are expected to last. These are represented by λ, the meeting rate and by δ, the rate at which productive matches break down. The searching partner also needs to take into account the distribution of types of the other partners looking for a match. Denote by α_i the proportion of type p_i's in the search pool, and by N_i the number of that same type in the pool. Hence,

$$\alpha_i = N_i \Big/ \sum_j N_j$$

Similarly, call γ_i the ratio of type p_i's in the entire economy and by L_i their respective number in the economy or type p_i labor force. Hence,

$$\gamma_i = L_i \Big/ \sum_j L_j$$

The partner searching for an opportunity to produce also has to take into consideration the wages offered in the market. These are given by c_{ij}, the compensation to type p_i when matched with type p_j. Finally, the partner must have expectations regarding the matching behavior of others. Denote by Π_{ij} the probability that a representative agent of type p_i is willing to match with type p_j. Anticipating rational expectations, this corresponds to type p_j's beliefs about type p_i's willingness to match with her. With all these considerations in mind, a partner of type p_i has to choose a probability π_{ij} of accepting to match, upon meeting type p_j. Notice that Π_{ij} defines how a representative agent behaves, whereas π_{ij} is the corresponding individual value.

Maximizing behavior implies that the value of search, in flow terms, is given by (in steady state)

$$\text{(1)} \qquad \forall i \in \{l, h\}, \quad rU_i = b_i + \lambda \sum_k \alpha_k \Pi_{ki} \max_{\pi_{ik} \in [0,1]} \{\pi_{ik}(M_{ik} - U_i)\}$$

When calculating her discounted expected value of search, type p_i knows that she receives income b_i while searching, but also considers the probability of a meeting (at rate λ per period of time). In case of an encounter, there is a probability α_k that the partner met is of type p_k. Type p_i believes there is probability Π_{ki} that type p_k is willing to match with her in which case she then has to decide whether to accept the match or continue search. She accepts to match if her surplus from the match is positive, randomizes if indifferent, and rejects it otherwise (if the partner met is not willing to match, type p_i continues to search). Equation (1) accounts for the fact that type p_i may encounter any one of two productivity types.

When matched with type p_j, type p_i receives compensation c_{ij} (per period of time). Matches break down at a rate δ per unit of time. Hence,

$$\forall (i, j) \in \{l, h\}, \quad r M_{ij} = c_{ij} + \delta(U_i - M_{ij}) \tag{2}$$

Output is divided between partners, so that

$$\forall (i, j) \in \{l, h\}, \quad c_{ij} + c_{ji} = f_{ij} \tag{3}$$

The wage negotiated is derived from the Nash bargaining solution, with disagreement points equal to the value of search for the respective partners. Hence, partners split the surplus from matching, where the surplus is defined as the value of a match less the value of search. This results in an even split of the surplus, since partners have equal bargaining powers. Therefore,

$$\forall (i, j) \in \{l, h\}, \quad M_{ij} - U_i = M_{ji} - U_j \tag{4}$$

Finally, the value functions depend on the proportion of the different types of partners in the search pool. In steady state, equality of the flows in and out of that pool implies

$$\forall\, i \in \{l, h\}, \quad \delta[L_i - N_i] = \lambda\left(\sum_k \alpha_k \Pi_{ik} \Pi_{ki}\right) N_i \tag{5}$$

The left-hand side of (5) represents the number of type p_i partners going back into the search pool (per period of time). The right-hand side represents the number of type p_i leaving that same pool. There are N_i of them and they meet at a rate λ. There is a probability α_k that a meeting is with type p_k and these encounters lead to matches with a probability $\Pi_{ik}\Pi_{ki}$, since a completed match requires the agreement of both partners.

We can now define an equilibrium:

DEFINITION. A matching equilibrium is comprised of value functions (U_i, M_{ij}), compensations (c_{ij}), skill distribution of searching partners (N_i), individual decision rules (π_{ij}), beliefs (Π_{ij}), such that $\forall (i, j) \in \{l, h\}$:

(i) The value functions correspond to maximizing behaviors by the partners, i.e., they solve the Bellman equations (1) and (2), with $\pi_{ij} = 1$ ($\in [0, 1]$, $=0$) if $M_{ij} - U_i > 0$ ($=0$, <0);

(ii) Upon matching, the bargaining outcome satisfies (3) and (4);
(iii) The numbers of each type in the search pool satisfy (5);
(iv) The beliefs are rational and there is consistency of individual and aggregate behavior, i.e., $\pi_{ij} = \Pi_{ij}$.

REMARK. The focus of this article is on steady-state pure-strategy equilibria. Chen (1999) proved existence of equilibrium, possibly in mixed strategies.

3. CHARACTERISTICS OF A MATCHING EQUILIBRIUM

Now that a matching equilibrium has been defined, it is possible to look at the general properties that all equilibria exhibit. The proofs are given in the Appendix.

PROPOSITION 1. *The value of search is strictly increasing in the partners' type.*

PROPOSITION 2. *Upon matching, partners do not split output, but rather equally split the output plus the differential in their value of search. In other words,* $\forall(i, j) \in \{l, h\}, c_{ij} = \frac{1}{2}[f_{ij} + r(U_i - U_j)]$.

A higher type can always follow the strategy of a lower type and get a higher value from it, because of higher output produced when matched. Therefore, the strategy actually chosen by the higher type has to result in a higher value of search. The second proposition is a direct consequence of the transferable utility assumption. It reflects the fact that the wage depends on the respective values of search, which "summarize" all the partners' equilibrium matching opportunities. Hence, partners must also be compensated for giving up matches with other potential types.

COROLLARY 1. *When matching, the higher productivity partner always retains a strictly bigger share of output.*

COROLLARY 2. *The wage that a higher productivity partner receives from matching with a particular type is strictly larger than the wage that a lower productivity partner receives from matching with that same type.*

Both corollaries are consequences of Proposition 2. Even though partners of different types split the match surplus equally, since the higher productivity partner has a higher value of search, she retains the larger share of output. Similarly, when a higher productivity partner matches with a given type p_k, not only does the match produce more output than if a lower productivity partner were to match with that type p_k, but also the higher type has a higher value of search. Hence, the more productive partner receives a bigger compensation. However, notice that it is not necessarily always the case that $c_{ij} > c_{ik}$, when $j > k$. This is because, even though more output is produced in the (i, j) match than in the (i, k) match, type p_j also has a higher value of search. So, an agent does not always want to match with the most productive type of partners, as opposed to the nontransferable utility case.

TABLE 1

THE POSSIBLE MATCHING EQUILIBRIA WITH TWO TYPES

Equilibrium Type	$(\Pi_{ll}, \Pi_{lh}, \Pi_{hh})$	Equilibrium Type	$(\Pi_{ll}, \Pi_{lh}, \Pi_{hh})$
BB	$(1, 1, 1)$	HL	$(0, 1, 0)$
LH	$(1, 0, 1)$	LN	$(1, 0, 0)$
HB	$(0, 1, 1)$	NH	$(0, 0, 1)$
BL	$(1, 1, 0)$	NN	$(0, 0, 0)$

Looking at all the possible matching patterns, the different equilibria are denoted in the following manner: The first letter represents who the low productivity type is willing to match with, and the second letter who the high productivity type is willing to match with. Focusing only on pure strategies, partners may match with low types only (L), high types only (H), both types (B), or no type (N). Note that if partners were not receiving any income while searching, the last possibility would not arise. Due to the Nash bargaining assumption, if type p_h is willing to match with type p_l, then the reverse must be true (i.e., $\Pi_{hl} = \Pi_{lh}$). Hence, the only possible equilibria are described in Table 1.

Since Burdett and Coles (1999) have already analyzed a similar setup at some length, I will not examine each type of equilibrium. Nonetheless, it is worth mentioning that multiple equilibria are also possible in this model. A sorting externality is the source of such multiplicity: Partners' matching decisions influence the distribution of types in the search pool, which in turn determines optimal strategies. However, the possible multiplicity of equilibria is not going to be the focus of the next section, where differences in matching behavior across countries will be explained through different labor market policies.

Before proceeding to the two applications, it is possible to find conditions reducing the number of equilibria to be considered. First, the equilibria where at least one of the types does not match at all (LN, NH, NN) will not arise, as long as the income received during search is "not too high." Second, if the production function is supermodular,[3] the HL equilibrium cannot arise. If, in addition, $b_h = b_l$, then the BL equilibrium cannot be supported either (matches where high types never match with other high types are precluded in this case).[4] Hence, if all the above conditions apply, at most three equilibria may arise BB, LH, and HB.

[3] In this context, supermodularity implies that $f_{ll} + f_{hh} > 2f_{lh}$. In other terms, if there are two high types and two low types, total output is maximized by matching partners of the same type.

[4] The HL equilibrium, where high types only match with low types and vice versa, corresponds to production functions where $f_{hh} \approx f_{lh}$ and $f_{ll} \approx 0$. In that case, low types do not have any other option but to match with high types, implying that high types are well compensated for accepting such matches. It may even be preferable for a high type not to work with another high type, since she would have to evenly split roughly the same match output, rather than retain most of it. This, of course, is only possible in the context of transferable utility, where a low type can induce a high type to match with her, by compensating the high type with a higher wage.

PROPOSITION 3

- *If* $\forall i \in \{l, h\}$, $f_{ii} > 2b_i$, *the LN, NH, and NN equilibria cannot exist.*
- *If f is supermodular, the HL equilibrium cannot exist. If, in addition,* $b_h = b_l$, *the BL equilibrium cannot exist.*

4. ARE MATCHING PATTERNS BETWEEN HETEROGENEOUS AGENTS RELEVANT TO THE STUDY OF LABOR MARKETS?

4.1. *Unemployment and Wage Dispersion.* There has been great interest recently among economists in explaining the different labor market outcomes in the United States and Europe, as witnessed by the abundant literature (Bentolila and Bertola, 1990; Bertola and Ichino, 1995; Bertola and Rogerson, 1997; Millard and Mortensen, 1997; Mortensen and Pissarides, 1999, just to name a few). All the explanations put forward rely on having different labor market policies in Europe and the United States. With the exception of Bertola and Ichino, however, the focus of these papers is only on one aspect of the divergence between American and European labor markets, namely unemployment differences.[5] Although these types of explanations have definite merits, it is possible, using the model, to take a different approach and investigate whether matching patterns between heterogeneous agents can simultaneously explain *both* the unemployment and wage dispersion differences across markets. In particular, it is often taken as given, without much rationalization, that wage-setting institutions in Europe result in more compressed wages. I want to propose an explanation that does not posit wage compression to induce high unemployment, but rather one where the two phenomena naturally arise together, i.e., I want to propose a rationale for endogenous wage compression. Hence, in the rest of this section, I will consider the possibility that the United States and Europe are in different equilibria, with the United States being in a low employment/high wage dispersion equilibrium and Europe in a high unemployment/low wage dispersion equilibrium.

First, one can find support for the claim that matching patterns are central for the simultaneous determination of wage and employment, by looking at the extent of under- and overeducation in labor market matches in various countries. Evidence on matching patterns is obtained by comparing the level of education required for a particular job with the education actually completed by the worker holding that position.[6] Denote a match as *proper*, if completed education is equal

[5] Note that a legally imposed minimum wage may also generate higher unemployment and lower wage dispersion. However, there is smaller wage dispersion *within type* in Europe, even after controlling for the usual skill proxies (education, experience). A minimum wage cannot explain smaller wage dispersion at high skill levels in Europe. Therefore, I do not pursue an explanation in this direction, but rather attempt to provide an explanation for endogenous wage compression.

[6] Following Hartog (2000), required schooling is typically measured in three different ways. The first method uses Job Analysis (JA) data. This involves the evaluation of the required level and type of education for the job titles in an occupational classification, by professional job analysts. The second method, uses Worker Self Assessment (WA) data, which consists of the worker specifying the education required for the job (as in the PSID). Finally, the information can be obtained from realized matches (RM), where the required education is derived from what workers usually have attained, i.e., the

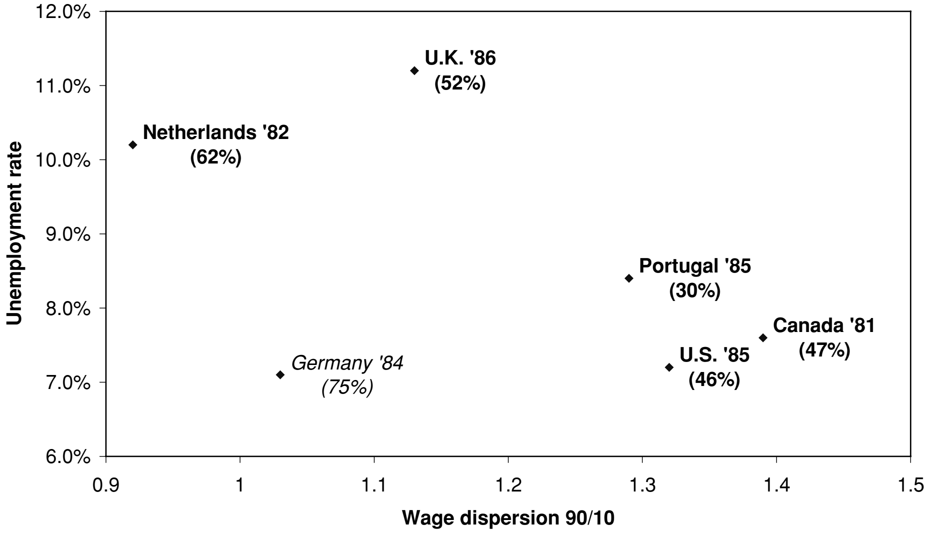

FIGURE 1

CROSS-COUNTRY COMPARISON (% OF PROPER MATCHES IN PARENTHESES)

to required education. Acemoglu (1999) finds that, in the United States, 46% of the workers sampled report proper matches.[7] Hartog (2000) reports that this number is 62% for the Netherlands, 52% for the United Kingdom, and 30% for Portugal, while Daly et al. (2000) and Vahey (2000) find that 75% and 47% of workers report proper matches in Germany and Canada, respectively. This can be combined with evidence on wage dispersion and unemployment in these countries. Figure 1 shows the proportion of proper matches, unemployment rate, and wage dispersion (measured as the log difference between the 90th and the 10th percentile of the wage distribution) for these six countries, in particular years.[8] The graph confirms that, across countries, a high incidence of proper matches is associated with both low wage dispersion and high unemployment (looking at average unemployment gives the same picture). Canada, the Netherlands, Portugal, the United Kingdom, and the United States fit that description exactly. Germany, which has the highest proportion of proper matches, also has the lowest wage

mean or the mode of that distribution. Notice that, when both methodologies were available, WA and JA provided similar results (Netherlands). Using one of these methodologies, it is possible to measure the incidence of (i) matches where workers are overeducated, (ii) matches where workers are undereducated, and (iii) proper matches, where workers have the correct education level for the job.

[7] This number is confirmed in Hartog (2000), Hartog and Oosterbeek (1988), and Sicherman (1991).

[8] The limited availability of data on proper matches dictated the countries and years reported. However, all the data falls in a relatively short time period (1981–86). Portugal was the only country for which data were available for several years between 1981 and 1986, and it exhibited roughly constant incidence of proper matches. The data on wage dispersion comes from the OECD Employment Outlook (1996) and the unemployment rate data from the Bureau of Labor Statistics database.

dispersion. However, it exhibits relatively low unemployment. This was pointed out in Nickell and Bell (1996), who underline the specificity of the German educational system. In Germany, two-thirds of the teenagers participate in an apprenticeship training system, receiving both classroom and on-the-job training. This can be expected to promote the formation of matches between an apprentice and her firm, therefore to reduce unemployment.

Given that partnership heterogeneity appears to be a key determinant of wage dispersion and unemployment, one can investigate which parameters give rise to the various matching patterns, and see if these parameters are consistent with American and European labor markets. Distinct matching patterns may be either due to different fundamental parameters or to different beliefs. Since it is recognized that labor market policies are very different in Europe and the United States, I will emphasize the first approach. In particular, I will focus on two types of policies: unemployment insurance ("UI") and firing costs ("FC"). By providing income while searching, UI may influence the decision of whom to match with. By having an impact on match stability, FC may also affect this same decision. I will first present simulations outlining which sets of parameter values give rise to particular equilibria. Then, I will present some evidence that, in line with the respective policies in place, actual parameters are consistent with European workers matching in equilibrium with a limited set of productivity types, and American workers with a larger set of types. In other words, the characteristics of the American and European labor markets lead to more homogeneity within European than American matches.

Let us first consider, with a simulation, how UI may affect the matching decision of heterogeneous workers. The technology parameters, skill distribution, and rate of time preference are fixed, while I assume that the search income is the sum of a fixed term and a term proportional to skill. Specifically, suppose that $b_i = b + \rho i$, $\rho \geq 0$.[9] The term b represents the value of leisure and/or home production, assumed not to be entirely proportional to skill, whereas the term ρi stands for unemployment benefits, which are typically proportional to wage, hence to skill.

One can see that LH equilibria are consistent with greater search income, whether due to higher b or to higher ρ, while BB equilibria can only be supported when partners receive little income while searching.[10] There is a trivial effect of including search income in the model: as b_l and b_h increase and become greater than output, partners may refuse to match (however, this can be done away by assuming that $2b_i < f_{ii} \forall i$, as in Figure 2). Besides this, search income has two effects on matching patterns. Both tend to make partners more "picky." First, when partners receive income while searching, the opportunity cost of matching—to both types—increases. But there is also a consideration that affects the formation of lh matches in particular. If b_h increases relative to b_l (i.e., if ρ increases), the differential in search value $U_h - U_l$ increases and lh matches become harder to sustain. This becomes clear by noticing that $c_{hl} - c_{lh} = r(U_h - U_l)$. Hence, as b_h

[9] That is, $b_l = b + \rho$ and $b_h = b + 2\rho$.

[10] *HB* equilibria are also consistent with high search income, when the fixed term b is high enough.

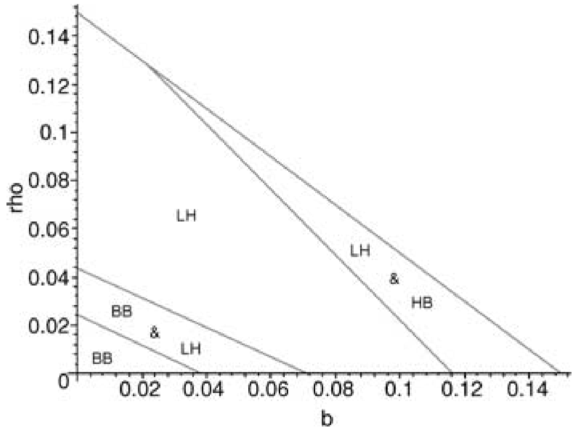

FIGURE 2

$\gamma_h = 1/3,\ r = 0.02,\ (f_{ll}, f_{lh}, f_{hh}) = (0.3, 0.5, 0.9),\ \lambda = 6,\ \delta = 0.6$

increases relative to b_l, c_{hl} must increase (for a given equilibrium) and it becomes more difficult for type p_l to compensate type p_h for accepting to match.[11]

Expected match duration is another factor that can affect the nature of the matching equilibrium. For that reason, I also simulate the effects of FC regulations on matching patterns. To that effect, the technology parameters, skill distribution, and rate of time preference are fixed once more, whereas the meeting and breakdown rates (λ, δ) are allowed to vary.

LH equilibria are consistent with higher values of λ/δ, whereas *BB* equilibria are associated with lower such ratios.[12] These results are intuitive, since a high meeting rate or a low breakdown rate justify high types being patient and waiting to meet other high types. Under such parameters, a low type cannot compensate a high type enough to accept a match with him. This is because $U_h - U_l$ is a decreasing (increasing) function of δ (λ). In other words, matching with the first partner occurs when it is not justified to wait for a better match (in the same spirit, and using unreported simulations, *LH* equilibria are associated with large proportions of high-skill workers and low discount rates).

Examination of UI regulations reveals that benefits tend to be more generous in Europe than the United States, both in terms of replacement rate and benefit duration (OECD, 1994).[13] As the simulations just revealed, this should lead to more homogeneity within matches in Europe than in the United States. In particular,

[11] Although their setup is very different from this one, the presence of UI may also affect the *composition* of jobs in Acemoglu and Shimer (1999).

[12] This example is carried out, assuming that $b_h = b_l = 0$. All the production functions simulated returned an equilibrium graph similar to Figure 3. The only way to have *HB* as an equilibrium is to choose f such that $f_{hh} + f_{ll} \gtrsim 2f_{lh}$ and $\gamma_h \geq \frac{1}{2}$.

[13] Up to 1985, Portugal was offering very few unemployment benefits, compared to European standards (OECD Database on Unemployment Benefit Entitlements and Replacement Rates). Portugal has also low benefit coverage rates among unemployed (OECD, 1994). The example of Portugal is

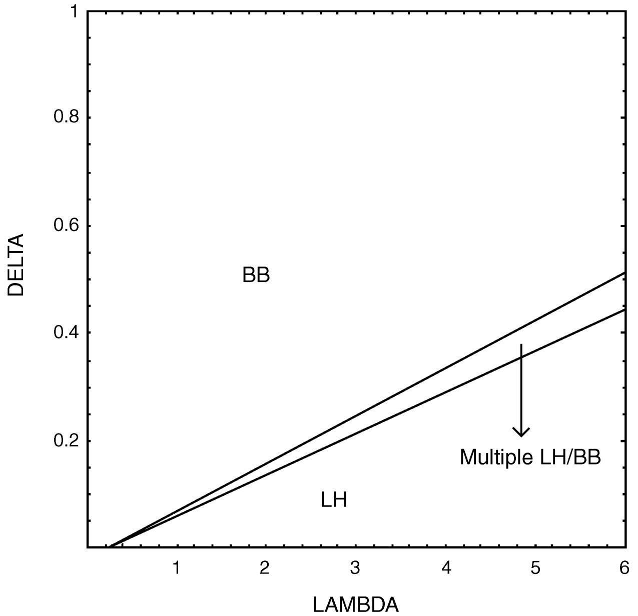

FIGURE 3

$\gamma_h = 1/3,\ r = 0.02,\ (f_{ll}, f_{lh}, f_{hh}) = (0.3, 0.5, 0.9)$

more generous benefits can be interpreted as a higher ρ in the model. The empirical evidence also supports the notion that match breakdowns are more frequent in the United States than in Europe. For example, Mortensen and Pissarides (1999) report that inflow rates into unemployment are two to eight times higher in the United States than in Europe. Further evidence from OECD (1994) shows that the average job tenure is greater in Europe, whereas the percentage of tenure of less than one year is greater in the United States. All of this indicates that matches break down at a lower rate in Europe than in the United States, which according to the model, promotes within-type matching.[14]

All the above considerations are compatible with the notion that partners match with a larger set of productivity types, or a greater percentage of the population, in North America than in Europe.[15] The latter has a higher incidence of proper

consistent with the model, however, since, in 1985, it exhibited low unemployment, high wage dispersion and a small incidence of proper matches.

[14] All the theoretical literature (Bentolila and Bertola, 1990; Delacroix, 2003; Hopenhayn and Rogerson, 1993; Millard and Mortensen, 1997; and Mortensen and Pissarides, 1999) finds that higher firing costs lead to lower rates of job separation. However, using a Mortensen–Pissarides (1994) framework, the effect on job creation (market tightness) is qualitatively ambiguous. Nevertheless, regardless of the quantitative effect of FC on market tightness, workers' and firms' meeting rates would be affected in opposite directions by FC, making one side more selective, but the other one less selective. Hence, the focus on lower breakdown rates as the primary effect of FC, in the context of this model.

[15] Also, the prevalence of advance notice for high-skill workers in Europe may increase the proportion of matches between high types at the expense of matches with low types.

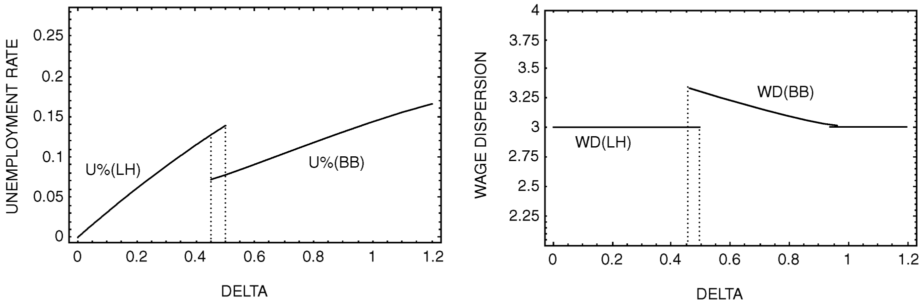

FIGURE 4

UNEMPLOYMENT AND WAGE DISPERSION: $\gamma_h = 1/3,\ r = 0.02,\ (f_{ll}, f_{lh}, f_{hh}) = (0.3, 0.5, 0.9),\ \lambda = 6$

matches, i.e., a higher proportion of matches with the required productivity type. In North America, however, matches tend to show more heterogeneity. Assuming different matching patterns across the ocean also fits the fact that higher unemployment is observed in Europe for all skill categories (and therefore is not only due to labor market policies affecting primarily the lesser skilled workers). It also implies that the duration of unemployment is higher in Europe than in the United States, as observed empirically.

In conclusion, the labor market policies in place in Europe promote within-type matching. What does this imply for unemployment and wage dispersion? In an economy, like the United States, where agents match with productivity types very different from theirs, this results in more matches for all types, and hence lower unemployment. In that economy, a high productivity type may accept to match with a lower type. However, there is an opportunity cost to the high type of matching with a low type. Hence, in the bargaining, the low type needs to compensate the high type to induce him to match. We know that, due to the transferable utility assumption, wages split output plus the differential in search values. When agents match with a larger set of types, this differential can become large in matches between agents that are quite different in productivity. Hence, one observes higher wage dispersion, because low types had to compensate higher types more for accepting to match. This is similar, in spirit, to the "opportunity cost effect" mentioned in Acemoglu (1997). These considerations can even explain higher within-type wage dispersion[16] in the United States, as reported in Bertola and Ichino (1995).

One can again simulate the model to verify that unemployment is higher and wage dispersion lower in an economy with homogenous matches only (LH equilibrium), than in an economy with heterogeneous matches (BB equilibrium). Keeping the same values for f, γ_h, and r and fixing $\lambda = 6$ (i.e., an average of two weeks between meetings), one can allow the breakdown rate to vary. Starting from $\delta = 0$ and increasing its value, the equilibrium changes from LH to BB, as in Figure 4. One can check that, as long as $\delta_{\text{Europe}} < \delta_{\text{U.S.}}$ are not too far apart, but yet generate different matching patterns, the resulting unemployment rate in

[16] That is, wage dispersion for workers of identical characteristics.

Europe ($U\%$ (LH)) is greater than the unemployment rate in the United States ($U\%$ (BB)). This graph also illustrates an important point. A BB equilibrium, by itself, does not guarantee both higher within-type wage dispersion *and* higher overall wage dispersion than in an LH equilibrium. Given all the restrictions imposed by Corollaries 1 and 2, there it still a possible wage pattern that produces higher within-type dispersion, but not higher overall dispersion—if p_l earns more when matched with p_h than with p_l and p_h earns less when matched with p_l than with p_h. In that case, $c_{ll} < c_{lh} < c_{hl} < c_{hh}$. However, one can show that this case can only arise for high values of δ, that is for very unstable matches (this can also be verified in Figure 4, where this happens only for $\delta > 1$). The intuition is that the more stable the matches, the better off type p_h is relative to type p_l. Conversely, as δ increases, $U_h \to U_l \to 0$, and the closer are types p_h and p_l from splitting output f_{lh} upon meeting, leading to greater within type, but not greater overall wage dispersion than in an LH equilibrium. Therefore, if δ is low enough, it is rather costly for type p_l to compensate type p_h for matching and hence, either $c_{lh}^{BB} < c_{ll}^{BB}$ or $c_{hl}^{BB} > c_{hh}^{BB}$ (depending on the nature of f).[17] Similarly, notice that a BB equilibrium does not *necessarily* guarantee lower unemployment than in an LH equilibrium, if the breakdown rate becomes too high. If matches are really unstable, partners match in every meeting, but job tenure is really low, implying high unemployment. Thus, for the argument to work, it has to be the case that δ_{Europe} and $\delta_{\text{U.S.}}$ are sufficiently far apart to generate different matching patterns, yet $\delta_{\text{U.S.}}$ cannot be too high, rendering partnerships too short lived and reducing the relative advantage of being a high type.

4.2. *Noise in Wages.* Another point can be made to further emphasize the importance of studying the matching patterns of heterogeneous agents. Abowd et al. (2001); (AKMT) show, using individual data on wages matched with firm data, that the combination of observed and unobserved individual characteristics and establishment effects, explain more of the French wage data than the American. In other terms, there is more noise in the United States than in the French wage data.[18]

From Proposition 2, we know that, for given productivity types p_i and p_j, the wage c_{ij} is not only a function of the productive characteristics of the two partners engaged in the match (f_{ij}), but also of their respective values of search, U_i and U_j. In equilibrium, type p_i optimally matches with a certain set of types. These

[17] $c_{lh}^{BB} < c_{ll}^{BB}$ if f is supermodular and $c_{hl}^{BB} > c_{hh}^{BB}$ is f is submodular.

[18] A referee pointed out that, in that survey, the French data were employer reported, whereas the United States data were partly based on employee-reported data, hence possibly more prone to measurement error (ME). Although this may be the case, there are reasons to believe that this should not negate the survey conclusions. Generally, employer data may also be characterized by ME. Also, some controls were of better quality in the United States than the French survey (education). In addition, the authors cleaned the data of outliers before proceeding to their analysis and found that their results were in accordance with some of the conclusions of Abowd et al. (1997), who use employer data for both France and the United States. Finally, Mellow and Sider (1983), running four regressions of wages on the standard right-hand side variables (depending on whether wages and RHS variables were employee or employer reported), find little difference in R^2. I thank the referee for that point.

matching opportunities affect her value of search, and hence the wage she can negotiate with type p_j. In conclusion, the wage c_{ij} depends on more than just f_{ij}. Because the values of search U_i and U_j reflect all the matching opportunities that p_i and p_j have in equilibrium, besides just matching with each other, c_{ij} also depends on the characteristics of members of type p_i's and p_j's matching sets (and even on which types the latter are matching with, and so on).

Applying this reasoning to France and the United States, one can expect more noise in wage data in the United States, after controlling for the partners' characteristics: if partners match with larger sets in the United States, then the characteristics of the match participants are less relevant in the wage determination. Indeed, we know from AKMT that regressions using only the characteristics of agents *within* the match have low predictive power.

5. CONCLUSION

An equilibrium model was developed where agents of different productivities have to decide which kind of partners to match with, when frictions make finding a partner a difficult and time-consuming process. The model was designed to replicate the salient features of a labor market. General characteristics of a matching equilibrium were underlined, emphasizing the importance of assuming transferable utility. The model was then applied to the issues of (i) wage dispersion and unemployment in Europe and the United States (including within-type wage dispersion) and (ii) the relationship between wage and firms' and workers' characteristics in France and the United States. It was shown that matching patterns may explain some of the differences between these labor markets. In fact, the model emphasized the need for more cross-country empirical research on matching behavior between agents of different productivities.

This framework is well adapted to the study of how labor market policies affect the composition of jobs (Acemoglu, 2001, is another recent example of that kind of endeavor). A natural extension would be to study how labor market policies interact with a skill-biased technological change to induce *changes* in unemployment and wage dispersion in the United States and Europe, as observed since the 1980s. In addition to wage dispersion, the difference in earnings mobility between the two labor markets could also be examined with such matching considerations.

APPENDIX

PROOFS OF PROPOSITIONS 1 AND 2 AND COROLLARIES 1 AND 2. Let us call $\Xi_i = \{j, M_{ij} \geq U_i\}$, type i's matching set. Equation (2) implies that $(M_{ij} - U_i) = \frac{c_{ij} - rU_i}{r+\delta}$. Hence, from (1), $rU_i = b_i + \frac{\lambda}{r+\delta}\sum_{k\in\Xi_i}\alpha_k(c_{ik} - rU_i)$. Equal split of the surplus implies that $c_{ij} - rU_i = c_{ji} - rU_j = f_{ij} - c_{ij} - rU_j$ and $c_{ij} = \frac{1}{2}[f_{ij} + rU_i - rU_j]$. Therefore, $rU_i = b_i + \frac{\lambda}{2(r+\delta)}\sum_{k\in\Xi_i}\alpha_k(f_{ik} - rU_i - rU_k)$. It is clear that $(M_{ij} - U_i)$ and $f_{ij} - rU_i - rU_j$ have the same sign, so that $j \in \Xi_i \Leftrightarrow f_{ij} - rU_i - rU_j \geq 0$. Let us now show that for any type and any subset S of $\{l, h\}$, $rU_i \geq b_i + \frac{\lambda}{2(r+\delta)}\sum_{k\in S}\alpha_k(f_{ik} - rU_i - rU_k)$.

Suppose that $\exists i \in \{l, h\}, \exists S \subset \{l, h\}, rU_i < b_i + \frac{\lambda}{2(r+\delta)} \sum_{k \in S} \alpha_k (f_{ik} - rU_i - rU_k)$ (H).

Then, $\sum_{k \in \Xi_i} \alpha_k (f_{ik} - rU_i - rU_k) < \sum_{k \in S} \alpha_k (f_{ik} - rU_i - rU_k)$. Neither $\Xi_i \backslash S \cap \Xi_i = \emptyset$, nor $S \backslash S \cap \Xi_i = \emptyset$, since each would contradict (H). Therefore, $\sum_{k \in \Xi_i \backslash S \cap \Xi_i} \alpha_k (f_{ik} - rU_i - rU_k) < \sum_{k \in S \backslash S \cap \Xi_i} \alpha_k (f_{ik} - rU_i - rU_k)$. However, the left-hand side is nonnegative and the right-hand side negative, a contradiction. Thus, $rU_h \geq b_h + \frac{\lambda}{2(r+\delta)} \sum_{k \in \Xi_l} \alpha_k (f_{hk} - rU_h - rU_k)$ and $r(U_h - U_l) \geq b_h - b_l + \frac{\lambda}{2(r+\delta)} \sum_{k \in \Xi_l} \alpha_k (f_{hk} - f_{lk} - r(U_h - U_l))$. Therefore, as long as $b_h \geq b_l$, $r[1 + \frac{\lambda}{2(r+\delta)} \sum_{k \in \Xi_l} \alpha_k](U_h - U_l) \geq b_h - b_l + \frac{\lambda}{2(r+\delta)} \sum_{k \in \Xi_l} \alpha_k (f_{hk} - f_{lk}) > 0$, implying that $U_h > U_l$.

Proposition 2 was established in the proof of Proposition 1. Using Proposition 2, Corollary 1 is proved by noting that $c_{ij} - c_{ji} = r(U_i - U_j)$ and Corollary 2 proved by calculating $c_{ik} - c_{jk} = \frac{1}{2}[f_{ik} - f_{jk} + r(U_i - U_j)]$. ■

Proof of Proposition 3. The equilibrium conditions can be summarized as follows (a proof may be provided upon request):

$$\Pi_{ll} = 1 \Leftrightarrow (r+\delta)(c_{ll} - b_l) > \lambda \alpha_h \Pi_{lh} (c_{lh} - c_{ll})$$
$$\Pi_{lh} = 1 \Leftrightarrow (r+\delta)(c_{lh} - b_l) > \lambda \alpha_l \Pi_{ll} (c_{ll} - c_{lh})$$
$$\Pi_{hh} = 1 \Leftrightarrow (r+\delta)(c_{hh} - b_h) > \lambda \alpha_l \Pi_{lh} (c_{hl} - c_{hh})$$

Type p_i agrees to match with type p_j iff $(r+\delta)(c_{ij} - b_i) > \lambda \alpha_{-j} \Pi_{i,-j} (c_{i,-j} - c_{ij})$, where $-j$ has the usual interpretation. That is, p_i matches with p_j if the gain from matching (as measured by the income received net of the value of search, taking into account the match impermanence), is greater than the expected gain from remaining in search and matching with the other type (p_{-j}).

An equilibrium where a particular type p_i does not match at all requires that $c_{ij} < b_i, \forall j \in \{l, h\}$. Thus, by assuming that $f_{kk} > 2b_k, \forall k \in \{l, h\}$, one ensures that such an equilibrium cannot arise (since $f_{kk} = 2c_{kk}$).

To see that an LH equilibrium cannot exist when the production function is supermodular, notice that such an equilibrium requires that $f_{hh} < 2rU_h$, $f_{ll} < 2rU_l$, and $f_{hl} > r(U_h + U_l)$. Subtracting the last condition from the sum of the first two, it must be the case that $f_{ll} + f_{hh} - 2f_{lh} < 0$.

Let us consider the conditions required for a BL equilibrium. In particular, $\Pi_{lh} = 1$ and $\Pi_{hh} = 0$ imply that $(r+\delta)(c_{lh} - b_l) > \lambda \alpha_l (c_{ll} - c_{lh})$ and $(r+\delta)(c_{hh} - b_h) < \lambda \alpha_l (c_{hl} - c_{hh})$. Subtracting the second condition from the first, one gets that $(r+\delta)(c_{lh} - c_{hh} + b_h - b_l) > \frac{1}{2}\lambda \alpha_l [f_{ll} + f_{hh} - 2f_{lh}]$. From Corollary 2, $c_{lh} < c_{hh}$. If f is supermodular and $b_h = b_l$, the left-hand side is negative, and the right-hand side is positive. ■

REFERENCES

Abowd, J., H. Finer, F. Kramarz, and S. Roux, "Job and Wage Mobility: An Analysis of the Dynamics of Employment Durations Using Matched Employee and Employer Data from the U.S. and France," Working paper, Cornell University, 1997.

———, F. Kramarz, D. Margolis, and K. Troske, "The Relative Importance of Employer and Employee Effects on Compensation: A Comparison of France and the United States," *Journal of the Japanese and International Economies* 15 (2001), 419–36.
Abraham, K., and S. Houseman, "Earnings Inequality in Germany," in R. Freeman and L. F. Katz, eds., *Differences and Changes in Wage Structure* (Chicago, IL: Chicago University Press, 1995).
Acemoglu, D., "Matching, Heterogeneity, and the Evolution of Income Distribution," *Journal of Economic Growth* 2 (1997), 61–92.
———, "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review* 89 (1999), 1259–78.
———, "Good Jobs versus Bad Jobs: Theory and Some Evidence," *Journal of Labor Economics* 19 (2001), 1–22.
———, and R. Shimer, "Efficient Unemployment Insurance," *Journal of Political Economy* 107 (1999), 893–928.
Albrecht, J., and S. Vroman, "A Matching Model with Endogenous Skill Requirements," *International Economic Review* 43 (2002), 283–305.
Bentolila, S., and G. Bertola, "Firing Costs and Labor Demand: How Bad is Eurosclerosis?" *Review of Economic Studies* 57 (1990), 381–402.
Bertola, G., and A. Ichino, "Wage Inequality and Unemployment: United States versus Europe," *NBER Macroeconomics Annual* 10 (1995), 13–66.
Bertola, G., and R. Rogerson, "Institutions and Labour Reallocation," *European Economic Review* 41 (1997), 1147–71.
Burdett, K., and M. Coles, "Marriage and Class," *Quarterly Journal of Economics* 112 (1997), 141–68.
Burdett, K., and M. Coles, "Long-Term Partnership Formation: Marriage and Employment," *The Economic Journal* 109 (1999), F307–F334.
Chen, F., "Bargaining and Search in Marriage Markets," Mimeo, University of Chicago, 1999.
Daly, M., F. Büchel, and G. Duncan, "Premium and Penalties for Surplus and Deficit Education. Evidence from the United States and Germany," *Economics of Education Review* 19 (2000), 169–78.
Delacroix, A., "Transitions into Unemployment and the Nature of Firing Costs," *Review of Economic Dynamics* (2003), forthcoming.
Hartog, J., "Over-Education and Earnings: Where We Are, Where Should We Go?" *Economics of Education Review* 19 (2000), 131–47.
Hartog, J., and H. Oosterbeek, "Education, Allocation and Earnings in the Netherlands: Overschooling?" *Economics of Education Review* 7 (1988), 185–94.
Hopenhayn, H., and R. Rogerson, "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy* 101 (1993), 915–38.
Katz, L., G. Loveman, and D. Blanchflower, "A Comparison of Changes in the Structure of Wages in Four OECD Countries," in R. Freeman and L. F. Katz, eds., *Differences and Changes in Wage Structure* (Chicago, IL: Chicago University Press, 1995).
Kremer, M., and E. Maskin, "Wage Inequality and Segregation by Skill," NBER Working paper 5718, 1996.
Krusell, P., L. Ohanian, J.-V. Rios-Rull, and G. Violante, "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica* 68 (2000), 1029–53.
Mellow, W., and H. Sider, "Accuracy of Response in Labor Market Surveys: Evidence and Implications," *Journal of Labor Economics* 1 (1983), 331–44.
Millard, S., and D. Mortensen, "The Unemployment and Welfare Effects of Labour Market Policy: A Comparison of the U.S. and U.K.," in D. Snower and G. de la Dehesa, eds., *Unemployment Policy: How Should Governments Respond to Unemployment?* (Oxford: Oxford University Press, 1997).
Mortensen, D., and C. Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61 (1994), 397–415.
———, and ———, "Unemployment Responses to Skill-Biased Technology Shocks: The Role of Labor Market Policy," *The Economic Journal* 109 (1999), 242–65.

Nickell, S., and B. Bell, “Changes in the Distribution of Wages and Unemployment in OECD Countries,” *American Economic Review Papers and Proceedings* 86 (1996), 302–8.

OECD Database on Unemployment Benefit Entitlements and Replacement Rates.

Organization of Economic Cooperation and Development, *Job Study* (Paris: OECD, 1994).

———, *Employment Outlook* (Paris: OECD, 1996).

Sattinger, M., “Search and the Efficient Assignment of Workers to Jobs,” *International Economic Review* 36 (1995), 283–302.

Shimer, R., and L. Smith , “Assortative Matching and Search,” *Econometrica* 68 (2000), 343–69.

Sicherman, N., “ ‘Over-education’ in the Labor Market,” *Journal of Labor Economics* 9 (1991), 101–22.

Vahey, S., “The Great Canadian Training Robbery: Evidence on the Returns to Educational Mismatch,” *Economics of Education Review* 19 (2000), 219–27.