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Pricing and signaling with frictions ☆

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Abstract

We study a market where each seller chooses the quality and price of goods and the number of selling sites. Observing sellers' choices of prices and sites, but not quality, buyers choose which site to visit. A seller's choices of prices can direct buyers' search and signal quality. A unique equilibrium exists and is separating. When the quality differential is large, the equilibrium implements the efficient allocation with public information. Otherwise, the quality of goods and/or the number of sites created is inefficient, due to a conflict between the search-directing and signaling roles of prices.

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1. Introduction

In this paper, we study a large market with directed search and signaling. On one side of the market, each individual chooses an investment that determines the quality of the good which is the individual's private information. Individuals enter a search market competitively to create trading sites for their goods and can choose the price to which they commit in trade. In this environment, a posted price can signal the quality of the good, as well as direct the search by individuals on the other side of the market. After matching and before deciding whether or not to trade, an individual's trading partner observes an imperfectly informative signal about the quality. We characterize the equilibrium in this market and evaluate its (in)efficiency. We also compare this equilibrium with the equilibrium under bargaining.

The features outlined above are common in many markets, where the match quality is partially realized after search and the remainder is discovered by experiencing (consuming) the good. One example is the labor market, where firms can create jobs that differ in amenities and working conditions. Workers can find out the quality of a particular job partly by visiting/interviewing with the firm and partly by working in the firm. Another example is the market for goods such as a piece of furniture. A seller can make the furniture differ in quality. A buyer can get information about the quality by visiting the store and inspecting the furniture but, even after that, the buyer may only be able to find the true quality by purchasing and using the furniture. Let us refer to the individuals who undertake the investment in quality and create trading sites as "sellers", and the individuals on the other side of the market as "buyers". To understand resource allocation in such markets, it is important to formally model how pricing decisions by privately informed sellers interact with search frictions. There has been little research on the interaction between these two elements, although there is a large literature on each separately (see the references later).

To study this interaction in a concrete way, we examine an economy with many sellers and buyers where each seller chooses whether to produce high-quality or low-quality goods. High quality is more costly to produce and yields higher utility to a buyer than low quality. However, we assume decreasing average cost so that when quality is public information, only high quality is produced in the equilibrium. A seller also chooses how many units to produce, and so the supply of goods is endogenous. However, as part of search frictions, a seller must incur the cost of creating a selling site for each unit of the good he produces. Selling sites can be located in places that are characterized by a price label, which we call "submarkets". Thus, a seller's pricing decision is to choose which submarket to enter to create trading sites for his goods. Each buyer chooses which submarket to enter after observing sellers' choices of submarkets and sites, but not quality. By entering a submarket with a particular price, an individual is committed to the price if trade occurs. Matching is bilateral inside each submarket. After being matched, a buyer receives a signal about the quality of the good and decides whether or not to trade. We assume that this signal has a sufficiently high probability of being accurate. Also, following the majority of the models in the search literature and the signaling literature discussed below, we assume that the pricing mechanism in each submarket is the most common one in reality, i.e., a single price for a good. Section 5 will discuss more elaborate pricing mechanisms.

We prove that there is a unique equilibrium, under reasonable restrictions on buyers' beliefs about the quality of goods in inactive submarkets. In the equilibrium, only one quality is produced. This quality can be high or low, depending on the differential between low and high quality. When the quality differential is large, only high quality is produced, and the equilibrium with private information implements the efficient allocation with public information. When the quality differential is at the intermediate level, high quality is still the only quality produced, but the equilibrium has excessive entry of trading sites relative to the social optimum. When the quality differential is small, only low quality is produced, and the equilibrium has deficient entry of trading sites relative to the social optimum.

These results are due to the two roles of a posted price. First, a posted price can potentially signal the quality of the good. Because a buyer decides whether or not to trade after receiving a signal about the quality of the good in the match, the buyer will not buy a low-quality good when its price exceeds the utility of the low-quality good. Thus, a high-quality seller is able to signal the high quality by entering a submarket with such a high price. Second, posted prices can direct search and internalize matching externalities, as modeled by Peters [20,21], Moen [19], Acemoglu and Shimer [1], Burdett et al. [5], and Shi [23], among others. When choosing which submarket to enter, individuals make a trade-off between the price and the matching probability, because they rationally expect that the matching probability in a low-price submarket will be relatively high for a site and low for a buyer, given the same quality. In this sense, prices direct individuals' search (entry) into submarkets. In the case of public information, directed search induces individuals to enter the market in such an amount that an individual's marginal social contribution to match formation is equal to the individual's share of the match surplus indicated by the price. Prices are efficient in this case because they compensate an individual with the marginal contribution to match formation.[1]

The two roles of a posted price may or may not conflict with each other, depending on whether or not the quality differential is below a threshold. If the quality differential is above the threshold, the price of a high-quality good is higher than the utility of the low-quality good, and there is no conflict between the two roles. In this case, the equilibrium under private information implements the social optimum under public information. If the quality differential is below the threshold, there is a conflict between the two roles of a posted price. To signal high quality, the price should be higher than the utility of the low-quality good, but such a high price exceeds the price for efficiently directing search for high-quality goods. If the quality differential does not go below the threshold by much, the need to signal quality dominates the need to efficiently direct search. In this case, only high-quality goods are produced in the equilibrium and the price is higher than the efficient level, which induces excessive entry of selling sites into the market. If the quality differential goes sufficiently below the threshold, the need to efficiently direct search dominates the need to signal quality. In this case, all sellers produce low-quality goods, and the equilibrium price directs search efficiently for low-quality goods.

We define the social welfare function as the sum of expected match surpluses of all individuals in the market, and measure the welfare loss from private information as the difference in welfare between the social optimum and the equilibrium. This welfare loss depends non-monotonically on the quality differential. When the quality differential is above a threshold, the welfare loss is zero because the equilibrium is efficient. When the quality differential falls below the threshold but not by much, the welfare loss arises from excessive entry of selling sites, and it increases as the quality differential decreases further. When the quality differential is sufficiently below the threshold, the welfare loss arises from the low quality in the market, and it decreases toward zero as the quality differential does so. The role of a posted price in directing search is important for this non-monotonic pattern of the welfare loss. If there were no need to direct search, then there would be no need for a high-quality seller to push price below the utility of

[1] Thus, the meaning of "price posting" in such a directed search environment differs from that in undirected search models, such as Burdett and Judd [4], where sellers post prices but a buyer is unaware of who is posting what price before the matching.

the low-quality good, and so the welfare loss would not increase as the quality differential decreases.

To further understand the two roles of the price posting mechanism, we use the social welfare function to compare the mechanism with bargaining. To simplify the analysis under bargaining, we focus on the case where the signal received by a buyer after matching reveals the quality of the good accurately. In this case, bargaining always induces a seller to produce high quality. However, since bargaining takes place after matches are formed, bargained prices cannot direct search and hence, they generically fail to internalize matching externalities. The entry of selling sites into the market is either inefficiently low if a seller's bargaining power is too low relative to the Hosios condition (Hosios [13]), or inefficiently high if a seller's bargaining power is too high relative to this condition. Whether price posting or bargaining is more efficient depends on a seller's bargaining power and the quality differential. If the quality differential is so large that the efficient price under price posting is able to signal high quality, then the equilibrium under price posting is socially efficient, which is clearly superior to the equilibrium under bargaining. When the quality differential is small enough to make the price-posting equilibrium inefficient, there exists an interval of intermediate values of a seller's bargaining power in which the bargaining equilibrium is superior to the price-posting equilibrium. We thus show that the comparison in efficiency in Acemoglu and Shimer [1] does not carry over to the economy with private information about quality, because each of the two pricing mechanisms may be relatively better at reducing different frictions.

The two main ingredients in our analysis are directed search and signaling. Each ingredient has been analyzed in the literature separately but rarely together. In the literature on prices as a signal of quality, search frictions are either absent (e.g., Milgrom and Roberts [18]), or not explicitly modeled as a matching process (e.g., Bester [3]). On the other hand, the growing literature on directed search cited earlier either omits private information or does not model signaling. Putting the two ingredients together not only captures important features of realistic markets, but also uncovers the potential conflict between the two roles of posted prices. In addition, we show that price posting dominates bargaining under some parameter values and is dominated by bargaining under other parameter values.

There are recent papers that have incorporated private information into models of directed search. Peters and Severinov [22] analyze a market in which sellers use auctions to direct the participation of buyers who have independent values of a good. Michelacci and Suarez [17], Guerrieri [11] and Guerrieri et al. [12] analyze directed search markets with private information. These papers all assume that the individuals who choose the pricing mechanism do not have private information. That is, private information gives rise to adverse selection in these papers, rather than signaling as in our model. Menzio [15] allows firms to use cheap talks to signal private information on the quality of the vacancies they want to fill, but he excludes price signaling by assuming that wages are determined ex post by Nash bargaining.

In terms of the mix between directed search and price signaling, the paper closely related to ours is Albrecht et al. [2], whose focus is on the role of the list price in the housing market. In their model, a seller of a house can be matched with multiple buyers who draw independent values of the house. If two or more buyers are willing to pay the list price or above, then the seller holds an auction to determine the winner and the price. The list price is assumed to be a binding commitment if and only if exactly one buyer is willing to pay the list price. In contrast, we assume that matches are bilateral, that a good has a common-value component to all buyers, and that there is full commitment to the posted price. These assumptions help us focusing instead on the potential tension between the search-directing and the signaling role of a posted price.

Let us contrast our work more specifically with the signaling literature without search frictions, e.g., Milgrom and Roberts ([18], *MR* henceforth). The main difference is the presence of search frictions in our model. Search frictions introduce a potential conflict between the search-directing and the signaling role of a posted price, and they give rise to an endogenous demand curve facing each seller. There are other differences between our model and MR. First, while sellers in MR try to separate intertemporally by charging an introductory price first and then another price in the next period, sellers in our model separate atemporally by exploring the trade-off between the price and the trading probability. Second, we introduce an informative signal (other than posted prices) about the quality of the good that a buyer receives after being matched and before deciding whether to trade. This signal adds an inspection component to the goods, and it helps proving the uniqueness of the separating equilibrium. In particular, the signal makes it not optimal for a low-quality seller to post a very high price, because a buyer will not buy the good at such a price after receiving the signal. Third, a seller in our model can choose the quality of the good, while the quality is exogenously given in MR. The endogenous quality is important for restricting buyers' beliefs out of the equilibrium and, hence, for proving that the unique equilibrium is a separating equilibrium. If the quality is exogenous, there can be a continuum of pooling equilibria, as shown in Section 5.2. Such pooling equilibria arise because of the need to direct search, as we alluded to in the earlier explanation for the non-monotonic welfare loss.

In Section 2, we describe the model environment and analyze the efficient allocation with public information. In Section 3, we define the equilibrium under private information, prove that the unique equilibrium is a separating equilibrium, and analyze the efficiency properties. We then compare this equilibrium with the one under bargaining in Section 4. In Section 5, we discuss several extensions. First, we discuss whether a buyer has incentive to incur a cost to acquire information about the quality, in contrast to the baseline model where a buyer receives such information at no cost. Second, we prove that when the quality produced by a seller is exogenously given, there is a non-empty parameter region in which a continuum of pooling equilibria exists. This demonstrates the importance of the quality choice for the uniqueness of the separating equilibrium in the baseline model. Third, we explore more elaborate pricing schemes and explain why they are much less common in practice than the flat price, although some of them can implement the efficient allocation. Finally, we discuss the alternative organization of the market in which buyers, instead of sellers, set up trading sites. Appendices A through C provide necessary proofs.

2. The model

2.1. Environment

To simplify the terminology, we describe a goods market, although the model also captures important aspects of the labor market. The economy has one period. There are a large number of identical buyers, whose mass is normalized to 1. A buyer wants to consume one unit of an indivisible good, which can differ in quality k. The utility to a buyer of consuming each unit of a good of quality k is k, and the utility of not consuming is zero. There are also a large number of sellers of mass 1. The supply of goods is determined endogenously. A seller chooses the quality of the good and can produce as many units as desirable, but the seller must sell each unit separately. The cost of producing a good of quality k is $\psi(k)$ and, in addition, there is a cost c of creating a selling site for each good as described later.[2] The value to a seller of selling a good at

[2] All costs and prices in this paper are measured in terms of utility.

price p is p, and the value of an unsold good is zero. To simplify the analysis, we consider two quality levels $k \in \{\lambda, 1\}$, where $\lambda \in (\underline{k}, 1)$ and $\underline{k}$ is defined later. The good with quality 1 is the high-quality good and the good with quality λ the low-quality good. The sellers producing these goods are called high-quality sellers and low-quality sellers, respectively.

The quality of a good is the seller's private information. However, after meeting the seller and before deciding whether or not to trade, a buyer receives a signal, z, about the quality of the good, which has three possible realizations, $\{\text{"true"}, \lambda, 1\}$. With probability $\alpha \in (0, 1]$, the signal is "true", which reveals the true quality of the good to the buyer. With probability $1-\alpha$, the signal is noise; that is, the realization of the signal in this case is $z = k$ with probability $1/2$ for each $k \in \{\lambda, 1\}$. If $\alpha = 1$, a good is a pure "search" good, since a buyer in this case knows the quality of the good immediately after meeting the seller. For all $\alpha \in (0, 1)$, there is positive probability that the buyer has to consume a good in order to find the true quality. In this case, a good has both a search component and an experience component. Thus, our environment encompasses the cases of search goods ($\alpha = 1$) and experience goods ($\alpha < 1$). We assume that quality is not verifiable by a third party. Thus, prices cannot depend on quality directly.[3]

Goods are traded bilaterally at trading sites. The number of trading sites is determined endogenously by free entry of sites.[4] The cost of setting up a site is $c \in (0, 1)$. In this paper, we consider an economy where sellers create trading sites (see Section 5 for an alternative setup). For a site to be operational, a seller is required to incur both the cost ψ to produce a good and the cost c to set up a site. Before describing the market, let us impose the following assumption on the costs and the precision of the signal:

Assumption 1. There exists $\underline{k} \in (0, 1)$ such that $\psi(\underline{k}) + c = \underline{k}$. The cost and the signal satisfy: (i) $0 < \psi(k) < k - c$ for all $k \in (\underline{k}, 1)$, (ii) $0 < \psi'(k) \leqslant \psi(k)/k$ for all $k \in (\underline{k}, 1)$, and (iii) $\alpha > \alpha_0 \equiv 1 - \psi(\lambda) - c$.

Part (i) requires the utility of consuming a quality-k good to exceed the sum of the costs in producing the good and creating a site for it. This condition is necessary for a quality-k good to generate a positive surplus to society. For this reason, we focus on the non-trivial case where $\lambda \in (\underline{k}, 1)$. In part (ii), monotonicity of ψ is a standard property. The assumption $\psi' \leqslant \psi/k$ says that the average cost of production is non-increasing in quality. This assumption is made to clearly illustrate the effect of private information on efficiency. It ensures that if quality is public information, only high-quality goods are produced in the social optimum and the equilibrium. If low-quality goods are produced in an equilibrium, it is an inefficiency generated by private information.[5] Part (iii) assumes that the signal is sufficiently informative. This allows us to establish the main results cleanly. Note that $\alpha_0 > 1 - \lambda > 0$ by (i).

[3] Although the special case $\alpha = 1$ illustrates most of the main arguments, we choose to analyze the model for the interval $\alpha \in (\alpha_0, 1]$ instead of only the point $\alpha = 1$. This analysis enables us to show that the main results are "continuous" in α; that is, they hold not only for pure inspection goods but also for goods with both inspection and experience components. This continuity is not automatically guaranteed, especially in the presence of private information. In a related model, Forand [8] endogenizes the precision of the signal about quality as a choice of the seller.

[4] Notice that we fix the number of sellers and allow each seller to choose the number of trading posts. An alternative way to endogenize the supply of goods/sites is to assume that there is free entry of sellers and each seller can maintain only one selling site. Because production and matching technologies have constant returns to scale, the two ways of modeling are equivalent.

[5] Even though there are only two quality levels, we restrict the derivative ψ' to satisfy $\psi' \leqslant \psi/k$ because we will establish existence of the equilibrium and study equilibrium properties for all $\lambda \in [\underline{k}, 1)$.

In the majority of the analysis here, the pricing mechanism will be price posting with directed search, but bargaining will also be analyzed in Section 4 for comparison. Under both mechanisms, a buyer chooses whether or not to trade after meeting a seller and receiving the signal about the quality of the good. With posted prices, buyers and sellers are committed to the posted price if they trade. With bargaining, buyers and sellers do not know the price before matching, although they can form expectations about the price.

To describe the trading environment, it is useful to think the market as a continuum of potential submarkets. Each submarket has a distinct label $p \in [0, 1]$ at the entrance. Individuals who participate in submarket p are committed to the price p. Individuals enter the market in two groups sequentially. First, sellers simultaneously choose which submarket to enter and how many sites to set up and, then, buyers choose which submarket to visit. As mentioned above, a seller must also produce a good and incur the setup cost for each site. Inside each submarket, buyers and sites are matched bilaterally. Different sites owned by the same seller are treated independently in this matching process. The matching technology is assumed to have constant returns to scale. Let θ denote the endogenous tightness in a submarket, i.e. the ratio of selling sites to buyers in the submarket. The matching technology in each submarket is such that the matching probability is $F(\theta)$ for a buyer and $F(\theta)/\theta$ for a site. A buyer in a match receives a signal about the quality of the good and decides whether to trade. If trade occurs, the price must be the one posted in the submarket. After the trade, the period ends.

The use of submarkets to describe the trading environment simplifies the explanation of how prices direct search. A submarket is a collection of selling sites and buyers who search for that price. The assumption that sellers choose whether to enter before buyers do allows sellers' decisions to affect buyers' entry decisions. For this reason, we say that sellers "post" prices. The entry decision and the assumption of random matching inside each submarket capture the idea that prices can direct search. Specifically, given the same quality of the good, a submarket with a high price is expected to have fewer buyers and more sites than does a submarket with a low price. We will formalize this dependence of the tightness on the price later in the definition of an equilibrium. Expecting this dependence, an individual makes a trade-off between the price and the matching probability when choosing which submarket to enter. For this trade-off to be non-trivial, we require the matching technology to satisfy the following standard assumption[6]:

Assumption 2. (i) $F(\theta) \in [0, 1]$ and $F(\theta)/\theta \in [0, 1]$ for all $\theta \in [0, \infty)$; (ii) for all $\theta \in (0, \infty)$, F is strictly concave and twice continuously differentiable, with $F > 0$, $F' > 0$ and $F - \theta F' > 0$; (iii) $F(\infty) = F'(0) = 1$, $F(0) = \lim_{\theta\to\infty} \theta F'(\theta) = 0$.

Part (i) requires the matching probabilities to lie in $[0, 1]$ for all tightness. Concavity and differentiability in (ii) simplify the analysis of the trade-off between prices and matching probabilities. Moreover, the assumptions $F' > 0$ and $F - \theta F' > 0$ require intuitively that, as the number of sites per buyer increases, the matching probability should strictly increase for a buyer and strictly decrease for a site whenever both probabilities are strictly less than one. Part (iii)

[6] We assume that the function F is exogenous, as in the formulations of directed search by Moen [19] and Acemoglu and Shimer [1]. Some other models of directed search derive the function F from the strategic game among a finite number of individuals and then take the limit when the number of individuals goes to infinity (e.g., Peters [21], Burdett et al. [5], and Shi [23]).

specifies the boundary conditions on the matching probabilities.[7] The following well-known matching functions satisfy the above assumption:

Example 2.1. Assumption 2 is satisfied by the so-called urn-ball matching technology, $F(\theta) = \theta(1 - e^{-1/\theta})$ (see Burdett et al. [5]), and by the generalized telephone matching function, $F(\theta) = (\theta^{-\rho} + 1)^{-1/\rho}$, where $\rho > 0$.

2.2. *Efficient allocation under public information*

To provide a reference to compare efficiency, let us examine the efficient allocation chosen by a social planner who faces the same search frictions as the market does. Because the quality of goods is a choice in this model, the social planner who chooses the quality knows the quality, and so the appropriate notion of efficiency is the efficient allocation under public information. For the market with quality-k goods, the planner chooses the fraction of buyers to be allocated to the market, denoted as $n_b(k)$, and the numbers of sites per buyer in the market, denoted as $\theta(k)$. These choices require that the number of sites to be created for quality-k goods be $n_b(k)\theta(k)$. The planner maximizes social welfare, which is the sum of net utility in the economy. Because a buyer is matched with a site of quality k with probability $F(\theta(k))$, total utility generated by quality-k goods is equal to $n_b(k)F(\theta(k))k$. The total cost of producing quality-k goods and creating sites for them is $n_b(k)\theta(k)[\psi(k) + c]$. Thus, social welfare generated in market k is $n_b(k)W(k, \theta(k))$, where

$$W(k, \theta) = F(\theta)k - \theta\big[\psi(k) + c\big]. \tag{2.1}$$

$W(k, \theta)$ is the social surplus per buyer generated by quality-k goods. The social planner maximizes $\sum_{k=1,\lambda} n_b(k)W(k, \theta(k))$, subject to the constraints: $n_b(1) + n_b(\lambda) = 1$, $n_b(k) \in [0, 1]$ and $\theta(k) \in [0, \infty)$ for $k \in \{1, \lambda\}$.

For any given k, the function $W(k, \theta)$ is strictly concave in θ under Assumption 2, and so it is maximized at $\theta = \theta^*(k)$ where

$$\theta^*(k) \equiv F'^{-1}\left(\frac{\psi(k) + c}{k}\right). \tag{2.2}$$

Under Assumptions 1 and 2, $\theta^*(k) \in (0, \infty)$ and $\theta^*(k)$ is strictly increasing; moreover, $W(k, \theta^*(k)) > 0$, and $\frac{d}{dk}W(k, \theta^*(k)) > 0$.[8] These properties imply that the efficient allocation requires that only high-quality goods be produced. To simplify the notation, let us denote $\theta^*_H = \theta^*(1)$ and $\theta^*_L = \theta^*(\lambda)$. We summarize the above results as follows:

Lemma 2.2. *When quality is public information, the efficient allocation requires only high-quality goods to be produced, i.e.* $n_b(1) = 1$, *and the number of sites created in the market for such goods to be* θ^*_H.

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[7] Note that the assumption $F'(0) = 1$ implies $\lim_{\theta\to 0}[F(\theta)/\theta] = 1$ by L'Hôpital's rule, the assumption $F(\infty) = 1$ implies $\lim_{\theta\to\infty}[F(\theta)/\theta] = 0$, and the assumption $\lim_{\theta\to\infty} \theta F'(\theta) = 0$ implies $F'(\infty) = 0$.

[8] Existence and uniqueness of $\theta^*(k)$ follow from the features that $F(\theta)$ is strictly concave, $F'(0) = 1$, $F'(\infty) = 0$, and $0 < \psi(k) < k - c$. The function $\theta^*(k)$ is strictly increasing because $F'(\theta)$ is strictly decreasing in θ and $[\psi(k) + c]/k$ is strictly decreasing in k. To prove that $W(k, \theta^*(k)) > 0$, substitute $(\psi + c)/k$ from (2.2) into (2.1) to obtain $W(k, \theta^*) = k[F(\theta^*) - \theta^* F'(\theta^*)]$. Because $\theta^* > 0$, part (ii) of Assumption 2 implies $W(k, \theta^*) > 0$. Finally, the envelope condition yields $\frac{d}{dk}W(k, \theta^*(k)) = F(\theta^*) - \theta^*\psi'(k)$. Because $\psi' \leqslant \psi/k$ by Assumption 1 and $c > 0$, then (2.2) implies $\frac{d}{dk}W(k, \theta^*(k)) > F(\theta^*) - \theta^* F'(\theta^*) > 0$.

Although the efficient allocation does not involve prices, for the discussion in later sections it is useful to define the efficient transfer from a buyer to a seller when the buyer receives a quality-k good. We call this transfer the "efficient price" of a quality-k good and denote it as $p^*(k)$. This price gives each site zero expected profit and induces the tightness $\theta^*(k)$ for quality-k goods. Given the tightness θ, a site is matched with probability $F(\theta)/\theta$, and so expected profit of a site is $\frac{F(\theta)}{\theta}p - \psi(k) - c$. Setting this expected profit to zero at $\theta = \theta^*(k)$ and using (2.2), we obtain the efficient price level in market k as

$$p^*(k) = G\big(\theta^*(k)\big)k, \quad \text{where } G(\theta) \equiv \frac{\theta F'(\theta)}{F(\theta)}. \tag{2.3}$$

Denote $p_H^* = p^*(1)$ and $p_L^* = p^*(\lambda)$. It can be shown that $p_H^* > p_L^*$.[9] The function $G(\theta)$ defined above is the marginal share of the number of matches contributed by the sites.[10] Because p/k is the share of the match value given to the site, the efficient price compensates a site by the site's marginal contribution to the match formation. This is the so-called Hosios [13] condition. We denote $\gamma = G(\theta_H^*)$ and refer to it as the seller's efficient matching share in the high-quality market. Note that $\gamma > \psi(1) + c$. Also, we use (2.2) to write the condition for θ_H^* equivalently as

$$\frac{F(\theta_H^*)}{\theta_H^*} = \frac{\psi(1) + c}{\gamma}. \tag{2.4}$$

3. Signaling and directing search with prices

In this section, we define and analyze the equilibrium in the economy with private information and price posting.

3.1. Equilibrium definition

We formulate a seller's entry decision as part of a signaling game. Let S be the set of all sellers and $s \in S$ be any arbitrary seller. Denote the number of sites for quality-k goods that seller s sets up in submarket p as $n_s(p,k)$, where $n_s : [0,1] \times \{\lambda, 1\} \to \mathbb{N}$ (including 0). Because buyers observe the total number of sites set up by a seller in a submarket, but does not observe the quality, a message sent by seller s is $m_s = \{\sum_k n_s(p,k)\colon\ p \in [0,1]\}$. The aggregate collection of messages across sellers is given by $M = \{m_s\colon\ s \in S\}$. Define by $\mathcal{M}$ the set of all possible aggregate messages. After observing M, buyers form (common) beliefs about the composition of the quality in each submarket and then choose which submarket to visit. Let $\mu : [0,1] \times \mathcal{M} \to [0,1]$ denote the buyers' beliefs, where $\mu(p, M)$ is the probability that an arbitrary site in submarket p offers a high-quality good. We will omit the dependence of the beliefs on M whenever there is no confusion. Given the belief μ, the expected value of a good to a buyer is denoted $\phi(\mu) = \mu + (1-\mu)\lambda$.

Individuals also form expectations about the number of buyers who will visit a submarket and, hence, about the tightness in the submarket. Because the tightness can depend on the belief μ, we denote the tightness in submarket p as $\theta_\mu(p)$, where $\theta_\mu : [0,1] \to \mathbb{R}_+$ and $\mathbb{R}_+$ is interpreted

9 Since $F'(\theta^*(k))k = \psi(k) + c$ by (2.2), then $p^*(k) = [\psi(k) + c]\theta^*(k)/F(\theta^*(k))$. The function $\theta/F(\theta)$ is increasing in θ by part (ii) of Assumption 2, and $\theta^*(k)$ is increasing in k as established above.

10 Notice that part (ii) of Assumption 2 ensures that $0 < G(\theta) < 1$ for all $\theta \in (0, \infty)$.

to include $+\infty$. For each p, $\theta_\mu(p)$ is the expected number of sites per buyer in submarket p. In submarket p, the matching probability is $F(\theta_\mu(p))$ for a buyer and $F(\theta_\mu(p))/\theta_\mu(p)$ for a site.

To define an equilibrium, let us derive the expression for an individual's payoff from entering submarket p. Consider first a buyer who visits submarket p. The buyer will be matched with a site with probability $F(\theta_\mu(p))$. After the match and before receiving a signal about the quality of the good, the buyer expects that the signal will be "true" with probability α and noise with probability $1-\alpha$. If the signal is "true", the buyer expects the revealed quality to be high with probability μ and low quality with probability $1-\mu$. If the revealed quality is high, the buyer will buy the good and obtain surplus $(1-p)$. If the revealed quality is low, the buyer will buy the good if and only if $\lambda > p$.[11] Thus, if the signal is "true", the buyer expects to obtain the surplus, $\mu(1-p)+(1-\mu)(\lambda-p)I(\lambda>p)$, where $I(x)=1$ if x is true and 0 otherwise. If the signal is noise, the buyer's posterior belief will be equal to the prior belief, in which case the buyer's expected value of buying the good will be $\phi(\mu)$ defined above. The buyer will buy the good in this case if and only if $\phi(\mu)>p$. Putting these cases together and taking into account the matching probability, we calculate the buyer's expected surplus from visiting submarket p as

$$D_\mu(p)=F\big(\theta_\mu(p)\big)\begin{Bmatrix}\alpha\big[\mu(1-p)+(1-\mu)(\lambda-p)I(\lambda>p)\big]\\ +(1-\alpha)\big[\phi(\mu)-p\big]I\big(\phi(\mu)>p\big)\end{Bmatrix}. \tag{3.1}$$

Let $D \geqslant 0$ denote the maximum of a buyer's expected payoff among all the submarkets, which we call the buyer's market payoff.

Now consider a seller who sets up a site with quality k in submarket p. The site is expected to be matched with a buyer with probability $F(\theta_\mu(p))/\theta_\mu(p)$. After the match, the buyer's signal will be "true" with probability α and noise with probability $1-\alpha$. When the signal is "true", the buyer will buy the good if and only if $k>p$. When the signal is noise, the buyer will buy the good if and only if $\phi(\mu)>p$. Thus, after meeting a buyer and before the buyer receives a signal, the seller expects to be able to sell the good with probability $\alpha I(k>p)+(1-\alpha)I(\phi(\mu)>p)$. The seller must incur the cost of producing the good, $\psi(k)$, and the cost of setting up the site, c, regardless of whether or not the good is sold. Hence, the seller's expected profit from producing a quality-k good and setting up a site in submarket p is

$$\pi_\mu(p,k)=\frac{F(\theta_\mu(p))}{\theta_\mu(p)}p\big[\alpha I(k>p)+(1-\alpha)I\big(\phi(\mu)>p\big)\big]-\psi(k)-c. \tag{3.2}$$

The total number of sites for quality-k goods in submarket p is

$$N(p,k)=\sum_{s\in S}n_s(p,k). \tag{3.3}$$

Denote the set of submarkets that have a positive number of sites for quality k as $P_k=\{p\in[0,1]\colon N(p,k)>0\}$. The set of submarkets that contain all sites is $P=P_\lambda\cup P_1$. Submarket p is said to be active if $p\in P$. We define an equilibrium as follows[12]:

Definition 3.1. A Bayesian Nash equilibrium consists of a buyer's market payoff $D\geqslant 0$, a collection of sellers' messages $M=\{m_s\colon s\in S\}$ with $m_s=\{\sum_k n_s(p,k)\colon p\in[0,1]\}$, beliefs $\mu(p,M)$, and the tightness function $\theta_\mu(p)$ that satisfy the following requirements:

[11] For convenience, we assume that a buyer buys a good only if the purchase makes a strictly positive surplus, i.e., only if $k>p$. This avoids the unnecessary complexity in the borderline case $k=p$.

[12] Throughout this paper we focus on symmetric equilibria where all site owners use the same strategy.

(i) Sellers' optimal choices and free entry of sites: for every (p,k) such that $N(p,k)>0$,

$$\max_{p'\in[0,1],\,k'\in\{\lambda,1\}} \pi_\mu\big(p',k'\big) \leqslant \pi_\mu(p,k)=0. \tag{3.4}$$

(ii) Buyers' optimal choices: $D=\max_{p\in P} D_\mu(p)$ and, for all $p\in[0,1]$,

$$D_\mu(p)\leqslant D \quad \text{and} \quad \theta_\mu(p)\leqslant +\infty, \quad \text{with complementary slackness.} \tag{3.5}$$

(iii) Consistent beliefs: for all $p\in P$,

$$\mu(p)=N(p,1)\Big/\sum_k N(p,k). \tag{3.6}$$

(iv) The number of buyers visiting all submarkets adds up to one:

$$\int\limits_{p\in P} \frac{1}{\theta_\mu(p)} \sum_k N(p,k)\,dp = 1. \tag{3.7}$$

Requirement (i) incorporates two conditions on the sellers. The first is the inequality in (3.4), which requires a seller's choice of the message m_s to maximize expected profit. If seller s chooses to enter submarket p with quality k (i.e., chooses $n_s(p,k)>0$), this choice should yield expected profit no less than other choices (p',k'). The second requirement in (i) is free entry of sites, given by the equality in (3.4). That is, expected profit of a site must be zero in every active submarket. Because of this condition, the number of sites chosen by an individual seller in an active submarket is indeterminate in this case, although the total number of sites in a submarket is determinate in the equilibrium.

Requirement (ii) requires that, among all active submarkets, a buyer should choose to visit the one that maximizes his expected payoff. Since he is free not to participate in the market, this expected payoff must be non-negative. Moreover, the complementary slackness condition in (3.5) imposes the following requirement for all $p\in[0,1]$. If some buyers visit submarket p, i.e., if $\theta_\mu(p)<+\infty$, then a buyer's expected payoff in the submarket must be equal to the buyer's market payoff; if $\theta_\mu(p)=+\infty$, i.e., if no buyer visits submarket p, a buyer's expected payoff in the submarket must be strictly less than the buyer's market payoff. Because this requirement is imposed for all $p\in[0,1]$, not just for $p\in P$, it is a refinement on the beliefs about the tightness function $\theta_\mu(p)$. Such a restriction on beliefs off the equilibrium is common in models of directed search, including those with perfect information.[13] The restriction determines the relationship between the tightness and the price that is needed for the trade-off between the two. It is important to note that (3.5) involves both the tightness and the belief about the quality of the good in a submarket. Thus, this restriction does not completely pin down the belief about quality in a submarket that is inactive in an equilibrium.

Requirement (iii) says that a buyer's belief about the probability that a good in any active submarket p is high quality should be equal to the actual fraction of high-quality goods in that submarket. Requirement (iv) is self-explanatory.

[13] For similar restrictions on beliefs in directed search models with perfect information, see Acemoglu and Shimer [1], Shi [24], and Menzio and Shi [16]. For such restrictions in models with imperfect/incomplete information, see Menzio [15], Gonzalez and Shi [9] and Guerrieri et al. [12]. These restrictions are justified by players' trembling. In the current model, suppose that a small measure $\varepsilon>0$ of site owners enter every submarket exogenously. The subsequent entry by visitors ensures that the queue length in every submarket p must satisfy (3.5). When $\varepsilon\to 0$, the equilibrium approaches the one defined here.

3.2. Equilibrium possibilities and restrictions on beliefs

We start with the following lemma (see Appendix A for a proof):

Lemma 3.2. *Given any beliefs, a seller will optimally choose not to create* (a) *a high-quality site in submarket $p < \psi(1) + c$ or $p < \lambda$,* (b) *a low-quality site in submarket $p \geqslant \lambda$,* (c) *any site in submarket $p \in [\lambda, \psi(1) + c)$ if this interval is non-empty.*

This lemma is intuitive. A seller chooses which quality to produce and which submarket to enter at the same time. When making these choices, a seller understands that once inside a submarket, the matching probability for a site and the price for the good are independent of the quality of the good. If the price in the submarket is lower than the low quality, a buyer matched with the site will buy the good regardless of the signal, in which case the seller's expected revenue from entering this submarket is independent of the quality of the good. Since a high-quality good is more costly to produce than a low-quality good, expected profit from entering such a submarket with a low-quality site is strictly higher than with a high-quality site, and thus it is not optimal to create a high-quality site in such a submarket. Also, a seller will not enter a submarket with a high-quality site if the price in the submarket does not cover the combined cost of the good and the site. Thus, for a seller to choose to produce a high-quality good, the price must be greater than or equal to both the low quality and the combined cost of a high-quality good and a site, as stated in (a). In a submarket where the price is higher than the low quality, a buyer who is matched with a low-quality seller will not buy the good when the signal reveals the quality, and so the seller's expected revenue for a low-quality site is no more than $(1 - \alpha)p$. Because the signal is sufficiently accurate, i.e. $\alpha \geqslant \alpha_0$, this expected revenue is strictly lower than the combined cost of producing a low-quality good and creating a site. Thus, it is not optimal for a seller to create a low-quality site in such a submarket, as stated in (b). Moreover, if the low quality is lower than the combined cost of a high-quality good and a site, results (a) and (b) imply that it is not optimal for a seller to create any site in a submarket where the price lies between the two levels.

Although Lemma 3.2 implies that there cannot be an equilibrium with the same submarket featuring the two quality levels – that is, there is no pooling equilibrium, it does not preclude the possibility that some submarkets with low-quality goods coexist with other submarkets with high-quality goods. Neither does it preclude the possibility that there are two or more active submarkets that provide the same quality. Moreover, Lemma 3.2 is not a statement about the beliefs on a seller's play "out of the equilibrium", an action that is not expected to take place in an equilibrium. For example, suppose that a submarket $\tilde{p} \geqslant \max\{\lambda, \psi(1) + c\}$ is expected to be inactive in an equilibrium, i.e., $\tilde{p} \notin P$, but a site is observed in submarket $\tilde{p}$. Arbitrary beliefs attached to the quality of this site can be consistent with Definition 3.1, because the formula in requirement (iii) on the consistency of beliefs does not apply to $p \notin P$. Even though it is not optimal for a low-quality site to enter submarket $\tilde{p}$, there is nothing inconsistent with the equilibrium definition or Lemma 3.2 if buyers believe that the site in submarket $\tilde{p}$ has low quality.

This arbitrariness in the beliefs about the quality in inactive submarkets implies that there can be a large number of equilibria and, in fact, a continuum of equilibria (see Appendix A). In particular, consider the singleton set $P = \{p_a\}$, where $\max\{\lambda, \psi(1) + c\} \leqslant p_a < 1$, together with the beliefs $\mu(p_a) = 1$ and $\mu(p) = 0$ for all $p \neq p_a$. Under such beliefs, all sites in a submarket other than p_a will be regarded as low quality. Given such beliefs, a seller who sets up a high-quality site in a submarket other than p_a will make a loss. In Appendix A, we prove that there is no gain to set up a low-quality site either, and so submarket p_a is the only active submarket. Moreover, there is a range of values of p_a each of which constitutes an equilibrium with the specified beliefs.

The beliefs that support this continuum of equilibria are not "reasonable" by the intuition criterion developed by Cho and Kreps [6]. Specifically, consider the belief $\mu(p) = 0$ for $p \geqslant \max\{\lambda, \psi(1) + c\}$ and $p \neq p_a$ in the above analysis. Suppose that a seller enters submarket p as a play out of the equilibrium. A buyer adhering to the belief $\mu(p) = 0$ will not visit submarket p. But it seems reasonable for the buyer to believe $\mu(p) = 1$ instead. The reason is that it is never optimal for a low-quality seller to make this play out of the equilibrium (see (b) of Lemma 3.2), but a high-quality seller can break even from such a play *if* he can induce buyers to believe that the site has high quality. If $\mu(p) = 1$, the seller's expected profit from a high-quality site in submarket p is $pF(\theta)/\theta - \psi(1) - c$. Because $p \geqslant \psi(1) + c$, there are some non-negative values of θ that yield zero expected profit to the site. Understanding a low-quality seller's sure loss and a high-quality seller's potential gain from this play off the equilibrium, a buyer's reasonable belief is $\mu(p) = 1$. Thus, we impose the following restriction on beliefs:

Restriction 1. $\mu(p) = 1$ for all $p \geqslant p_0$, where $p_0 \equiv \max\{\lambda, \psi(1) + c\}$.

Moreover, since entering submarket $p < \lambda$ is never optimal for a high-quality site (part (a) of Lemma 3.2) but it may break even for a low-quality site, we impose:

Restriction 2. $\mu(p) = 0$ for all $p < \lambda$.

This restriction is in the spirit of forward induction (Mas-Colell et al. [14, p. 293]): buyers ask themselves what could have possibly happened at the entry stage, assuming that sellers have behaved rationally in their decision on the quality of the good.[14] Thus, endogenizing the quality helps restricting beliefs on out-of-equilibrium plays.

Restrictions 1 and 2, together with Lemma 3.2, imply that there are only three possibilities of an equilibrium: (i) some submarkets $p \geqslant p_0$ are active with high-quality goods, (ii) some submarkets with $p < \lambda$ are active with low-quality goods, and (iii) both types of submarkets are active. In all three cases, $P_1 \cap P_\lambda = \varnothing$, and $\mu(p) \in \{0, 1\}$ for all $p \in [0, 1]$.

3.3. Characterization of sellers' optimal choices

We analyze a seller's optimal choice of the price first and then of the quality. Consider first the optimal choice of the price by a seller who chooses high quality. By the analysis in the previous subsection, the quality choice comes with restrictions on the set of feasible prices. Specifically, the seller cannot enter any submarket where the price is below p_0, where p_0 is defined in Restriction 1. Moreover, under Restriction 1, $\mu(p) = 1$ for all $p \geqslant p_0$; that is, different choices of $p \geqslant p_0$ do not trigger buyers to change their beliefs. This implies that, conditional on choosing $p \geqslant p_0$, the seller's optimal choice maximizes expected profit from setting up a high-quality site under the belief $\mu(p) = 1$.[15] In the direct formulation of this maximization problem, the objec-

[14] One can imagine that sellers also post the following speech: "I have entered submarket $p < \lambda$. Notice that I am better off entering this submarket producing low rather than high quality".

[15] This is an elimination of dominated strategies (see Milgrom and Roberts [18]).

tive function is the seller's expected profit and the constraint is that the price and the implied tightness together give a visiting buyer the market payoff D. However, if it is optimal for a seller to enter submarket p, then the expected profit to the seller must be zero in an equilibrium, as required by (i) of Definition 3.1. Thus, we can characterize the seller's optimal choice by solving the dual problem in which the seller's choice maximizes a buyer's expected surplus subject to the constraint that the tightness of the submarket gives the seller zero expected profit. That is, a high-quality seller's optimal choice solves

$$D_1 \equiv \max_{p \geqslant p_0} \{F(\theta(p))(1-p)\text{: } \theta(p) \text{ satisfies } \pi(p,1)=0\}$$
$$\text{where } \pi(p,1) = p\frac{F(\theta(p))}{\theta(p)} - \psi(1) - c. \tag{3.8}$$

Here we have substituted $\mu(p)=1$ and $\phi(1)=1>p$ into (3.1) and (3.2). Denote the solution to the above problem as p_1 and the induced tightness as θ_1. Recall $\theta^*(k)$ defined in (2.2) and $\theta_H^* = \theta^*(1)$. Appendix A shows that the triple (θ_1, p_1, D_1) satisfies

$$\theta_1 \geqslant \theta_H^*, \qquad F(\theta_1)/\theta_1 \leqslant [\psi(1)+c]/p_0,$$
$$p_1 = \frac{[\psi(1)+c]\theta_1}{F(\theta_1)}, \qquad D_1 = F(\theta_1) - \theta_1[\psi(1)+c], \tag{3.9}$$

where the two inequalities hold with complementary slackness. The optimal choice p_1 is either at the corner p_0, in which case $\theta_1 > \theta_H^*$, or interior, in which case $\theta_1 = \theta_H^*$. If $p_1 > p_0$, then $p_1 = p_H^* = p^*(1)$, where $p^*(\cdot)$ is given in (2.3). Under Assumptions 1 and 2, it is easy to verify that the conditions in (3.9) define a unique triple (p_1, θ_1, D_1).

Analogously, we can characterize a low-quality seller's optimal decision on which submarket to enter. By the analysis in the previous subsection, a low-quality seller will choose to enter submarket $p < \lambda$. Under Restriction 2, entering any submarket $p < \lambda$ does not change the belief $\mu(p) = 0$. Thus, the choice p $(< \lambda)$ must maximize expected profit from setting up a low-quality site there under the belief $\mu(p) = 0$. Because $\phi(0) = \lambda > p$ in this case, we can formulate the seller's dual problem as

$$D_\lambda \equiv \max_{p<\lambda} \{F(\theta(p))(\lambda-p)\text{: } \theta(p) \text{ satisfies } \pi(p,\lambda)=0\}$$
$$\text{where } \pi(p,\lambda) = p\frac{F(\theta(p))}{\theta(p)} - \psi(\lambda) - c. \tag{3.10}$$

Denote the solution to the above problem as p_λ and the induced tightness as θ_λ. The triple $(\theta_\lambda, p_\lambda, D_\lambda)$ satisfies

$$\theta_\lambda = \theta_L^*, \qquad p_\lambda = p_L^*, \qquad D_\lambda = \lambda[F(\theta_\lambda) - \theta_\lambda F'(\theta_\lambda)], \tag{3.11}$$

where $\theta_L^* = \theta^*(\lambda)$ as defined in (2.2) and $p_L^* = p^*(\lambda)$ as defined in (2.3). Note that the optimal choice in this case is always interior, i.e., $p_\lambda < \lambda$, because $p_\lambda/\lambda = G(\theta_\lambda) \in (0,1)$. The conditions in (3.11) define a unique triple $(p_\lambda, \theta_\lambda, D_\lambda)$.

Now let us turn to a seller's optimal choice of quality. This optimal choice must maximize a visiting buyer's expected surplus; otherwise no buyer will visit the seller. From the above analysis, we know that if the seller produces high quality, the optimal submarket to enter is p_1 $(\geqslant p_0)$, where the associated tightness is θ_1 and the expected surplus to a visiting buyer is D_1. Similarly, if the seller produces low quality, the optimal submarket to enter is p_λ $(< \lambda)$, where the associated tightness is θ_λ and the expected surplus to a visiting buyer is D_λ. Thus, the seller's

Table 1
The unique equilibrium under price posting.

Cases	Existence region	Quality of goods	Posted prices	Market tightness	Welfare level
Case H1	$\underline{k} \leqslant \lambda \leqslant \gamma$	high	γ	θ_H^*	$W(1, \theta_H^*)$
Case H2	$\gamma < \lambda < \lambda_c$	high	λ	θ_1 $(> \theta_H^*)$	$W(1, \theta_1)$
Case L	$\lambda_c < \lambda \leqslant 1$	low	$p_L^* < \gamma$	θ_L^* $(< \theta_H^*)$	$W(\lambda, \theta_L^*)$

optimal choice of quality satisfies: $k = 1$ if $D_1 > \max\{D_\lambda, 0\}$, $k = \lambda$ if $D_\lambda > \max\{D_1, 0\}$, $k \in \{1, \lambda\}$ if $D_1 = D_\lambda > 0$, and $k = \varnothing$ if $\max\{D_1, D_\lambda\} \leqslant 0$. In the last case, trade is shut down because a buyer gets a lower payoff from participating in the market than staying out of the market.

3.4. Equilibrium and (in)efficiency

By construction, both D_1 and D_λ are non-negative. If $\max\{D_1, D_\lambda\} > 0$, then the market is active. Thus, an equilibrium with an active market is one of the following three cases: (a) if $D_1 > \max\{D_\lambda, 0\}$, buyers will visit submarket p_1 but not submarket p_λ, in which case submarket p_1 (with high-quality goods) is the only active submarket in equilibrium; (b) if $D_\lambda > \max\{D_1, 0\}$, buyers will visit submarket p_λ but not submarket p_1, in which case submarket p_λ (with low-quality goods) is the only active submarket in equilibrium; (c) if $D_1 = D_\lambda$ (> 0), the equilibrium is the borderline case where submarkets p_1 and p_λ are both active in equilibrium. We focus on the two generic cases, (a) and (b).[16] Let us refer to case (a) as the *high-quality equilibrium*, and to case (b) as the *low-quality equilibrium*. In both cases (a) and (b), it is straightforward to verify that requirements (i)–(ii) in Definition 3.1 are satisfied by construction of (p_1, θ_1, D_1) and $(p_\lambda, \theta_\lambda, D_\lambda)$, where $D = \max\{D_1, D_\lambda\}$. Requirements (iii)–(iv) are satisfied because P_1 and P_λ are singletons.

To determine an equilibrium amounts to finding the parameter region for each case above. In Appendix A, we carry out this task. In addition, we compute the welfare level which, as before, is defined as total expected utility in the economy. Because a seller makes zero expected profit from every site, welfare is equal to a buyer's expected surplus, D. Moreover, because only one submarket is active in the equilibrium, a buyer's expected surplus is equal to $W(k, \theta)$ defined by (2.1), where k is the quality, and θ the tightness, in the active submarket. Recalling the notation $\gamma = G(\theta_H^*)$, where G is defined in (2.3), we summarize the results in the following proposition:

Proposition 3.3. *Consider the economy with price posting and directed search. Maintain Assumptions* 1 *and* 2*, and Restrictions* 1 *and* 2*. The equilibrium and its welfare properties are summarized in Table* 1*, with the following specifics*: (i) *The market is always active in the equilibrium.* (ii) *There exists a $\lambda_c \in (\gamma, 1)$ such that the equilibrium is unique for all $\lambda \neq \lambda_c$. The high-quality equilibrium occurs when $\lambda < \lambda_c$, and the low-quality equilibrium occurs when $\lambda > \lambda_c$.* (iii) *When $\lambda \leqslant \gamma$, the equilibrium is socially efficient.* (iv) *When $\lambda > \gamma$, the equilibrium is socially inefficient. If $\gamma < \lambda < \lambda_c$, the equilibrium provides the efficient quality but an excessive number of sites. If $\lambda_c < \lambda \leqslant 1$, the quality and the number of sites are both deficient in the equilibrium.*

[16] The case $D_1 = D_\lambda$ occurs only when $\lambda = \lambda_c$, as proven in Appendix A.

The key to understanding the existence and welfare properties of the equilibrium listed in Table 1 is to understand the roles played by prices in this economy. There are two potentially conflicting roles of prices in our setup, directing search and signaling quality. The first role of posted prices is to direct individuals' search and, in our setup, direct individuals' entry into the submarkets. That is, because individuals choose which submarket to enter knowing all prices in all submarkets, their entry decision makes a trade-off between price and market tightness. Given the same quality of goods, a submarket with a higher price has higher tightness, i.e., a larger number of sites per buyer. For a seller, a relatively high price yields higher profit from trade, but a lower matching probability. For a buyer, a relatively high price yields a lower surplus from trade, but a higher matching probability. If an individual chooses to enter a submarket, the combination of the price and the tightness in the submarket must maximize the individual's expected payoff. This trade-off is made explicit by requirements (i) and (ii) in the equilibrium Definition 3.1, and the implied relationship between the tightness and the price is implemented as a constraint in the maximization problems (3.8) and (3.10).

If quality is public information, this role of posted prices in directing search generates such tightness in each submarket that internalizes the externalities caused by individuals' entry decision. This implication of directed search for efficiency has been examined in recent literature (see the introduction for references). Intuitively, when a seller chooses to enter a submarket to set up a site, the entry marginally increases the number of sites per buyer in the submarket. This increase in the tightness causes a negative externality to other sellers who enter the same submarket, by reducing their matching probability, and a positive externality to all buyers who enter that submarket, by increasing their matching probability. Because a seller is more likely to increase the number of sites in a submarket with a high price of the good than with a low price, the increase in the tightness is compensated by the higher price of the good. Similarly, a buyer's entry into a submarket generates two opposite externalities and the resulting decrease in the tightness caused by a buyer's entry is compensated by a lower price of the good in the submarket. Because sellers and buyers face the same combination of the price and tightness, there cannot be a gain in efficiency from further entry by either side. That is, buyers and sellers are compensated according to their marginal contributions to the social value created in the market, and so the externalities are internalized.

If quality is private information, posted prices also perform the role of signaling quality. Specifically, a price greater than or equal to p_0 reveals high quality, while a price lower than λ reveals low quality. Thus, the relative quality λ acts as a constraint on a seller's choice of the submarket. This "signaling constraint" is not binding when λ does not exceed the efficient price in the submarket that a high-quality site would want to enter, γ. This is Case H1 in Table 1, in which the equilibrium with private information implements the socially efficient outcome with public information. However, when $\lambda > \gamma$, the signaling constraint binds and the socially efficient outcome cannot be implemented by an equilibrium. A high-quality site would want to enter submarket γ that compensates the entry decision efficiently, but doing so would make it profitable for low-quality sellers to mimic and, hence, would induce buyers to view the site as low quality. This binding constraint renders the equilibrium socially inefficient.[17] Thus, if the quality differential between goods is large enough, prices can efficiently play both their search-directing and signaling roles; if the quality differential is small, prices cannot perform both roles efficiently.

[17] Note that because $\gamma > \psi(1) + c$, then $\lambda \leqslant \gamma$ implies $p_0(\lambda) \leqslant \gamma$. If $\lambda > \gamma$, $p_1(\lambda) = p_0(\lambda) = \lambda$, as listed in Case H2 in Table 1.

When the equilibrium is inefficient, the inefficiency is in the form of either inefficient entry of sites or inefficient quality, depending on the relative strength of the two roles of prices. If the relative quality of the low-quality good does not exceed γ by a large margin (i.e., if $\gamma < \lambda < \lambda_c$), the implicit cost generated by the binding signaling constraint is not high. In this case, it is worthwhile for a seller to produce high-quality goods and entering submarket λ to signal high quality. This is Case H2 in Table 1. Because the price is inefficiently high in this case, there is excessive entry of sites into submarket λ in the equilibrium. In contrast, if the relative quality of the low-quality good exceeds γ by a large margin (i.e., if $\lambda_c < \lambda < 1$), it would be too costly for a seller to enter submarket λ to signal high quality, because the resulting probability of trade would be too low. The optimal choice of a seller in this case is to produce low-quality goods and enter submarket p_L^*, which directs search efficiently given the low quality. This is Case L in Table 1. The production of low-quality goods in this case is a sharp contrast with the efficient allocation which never features such production. Because of the low quality, there are fewer sites entering the market in this case than in the efficient allocation.

We can measure the inefficiency created by private information as $(W^* - W)$, where W^* is the welfare level in the social optimum and W in the equilibrium. This inefficiency depends non-monotonically on the relative quality, λ. When λ is small, the equilibrium is efficient, and so inefficiency stays at zero as λ increases, as long as $\lambda < \gamma$. When λ lies in the interval (γ, λ_c), an increase in λ increases inefficiency by making signaling more difficult. When λ increases further into the interval $(\lambda_c, 1)$, the sellers start to produce low-quality goods. The inefficiency created by the low quality *decreases* as the relative quality λ increases. In the limit $\lambda \to 1$, inefficiency becomes zero again. Thus, inefficiency is the highest at $\lambda = \lambda_c$.

4. Comparison between price posting and bargaining

We consider bargaining as an alternative pricing mechanism and contrast the efficiency properties under the two mechanisms. The purpose is to illustrate further how the potential tension between the two roles of posted prices affects efficiency. Since we will use the Nash bargaining formula to solve for the price in a match, there may be potential inconsistency between the Nash bargaining outcome and the outcome of sequential bargaining under one-sided private information. To eliminate this inconsistency, in this section we focus on the case where the signal about the quality of the good is always accurate, i.e., $\alpha = 1$.

4.1. Equilibrium with bargaining

Suppose that the price of a good is determined by bargaining after a buyer meets a seller and observes the quality of the good. Because prices are not posted before matching, they do not direct search in this environment. Instead, there is just one market, where all sites face the same matching probability. Let θ now denote the number of sites per buyer in the entire market. A buyer visiting the market is matched with a site with probability $F(\theta)$, and a site is matched with a buyer with probability $F(\theta)/\theta$. The price of the good in a match is determined by Nash bargaining. As mentioned above, we focus on the case where the buyer in a match observes the quality of the good before bargaining. Because the costs of producing the good and creating a site are sunk at the time of bargaining, the seller in bargaining is interested in the price p, and the buyer in the surplus $(k - p)$. Assume that the bargaining power of the seller is $\sigma \in [0, 1]$. Nash bargaining solves: $\max_{p\in[0,k]} p^{\sigma}(k - p)^{1-\sigma}$, and the solution is $p = \sigma k$.

A seller chooses the quality and the number of sites to create, taking market tightness as given. Low-quality goods are never produced in the equilibrium under bargaining. The reason is that the average cost of production is non-increasing in quality and so, whenever a low-quality site can break even, a high-quality site can make strictly positive profit. To demonstrate this result, note first that expected net profit of a quality-k site is $\frac{F(\theta)}{\theta}\sigma k - \psi(k) - c$. Such profit must be zero if the market contains quality-k goods in the equilibrium; i.e., $F(\theta)/\theta = [\psi(k) + c]/(\sigma k)$. If a low-quality good is produced in the equilibrium, then $F(\theta)/\theta = [\psi(\lambda) + c]/(\sigma\lambda)$. This implies $F(\theta)/\theta > [\psi(1)+c]/\sigma$, because $[\psi(k)+c]/k$ is strictly decreasing in k. That is, expected profit of a high-quality site is strictly positive in this case, which is inconsistent with the equilibrium. Thus, with bargaining, the market contains only high-quality goods.

The seller's bargaining power determines whether or not the market is active in the equilibrium. If $\sigma \leqslant \psi(1) + c$, expected net profit of a high-quality site is strictly negative for all $\theta > 0$, in which case the market shuts down. If $\sigma > \psi(1) + c$, high-quality goods are produced in the equilibrium. Denote θ_b as the tightness of the market in this case. Then, the zero-profit condition for a site yields

$$F(\theta_b)/\theta_b = \big[\psi(1) + c\big]/\sigma. \tag{4.1}$$

The expected surplus for a buyer visiting the market is $(1 - \sigma)F(\theta_b)$. We can compare this equation with the counterpart in the efficient allocation, (2.4). Because the function $F(\theta)/\theta$ is decreasing, the comparison reveals that $\theta_b > \theta_H^*$ if $\sigma > \gamma$, and $\theta_b < \theta_H^*$ if $\sigma < \gamma$. We summarize the results with bargaining as follows:

Proposition 4.1. *Suppose that buyers observe the quality of the good in their match and that the price is determined by bargaining after this observation. There is a unique equilibrium. If $\sigma \leqslant \psi(1) + c$, the market shuts down. If $\sigma > \psi(1) + c$, the market is active and only high-quality goods are produced. In this case, the number of sites in the equilibrium is deficient (i.e. $\theta_b < \theta_H^*$) if $\sigma < \gamma$, excessive if $\sigma > \gamma$, and efficient if $\sigma = \gamma$.*

The efficiency requirement, $\sigma = \gamma$, is the so-called Hosios condition in the environment with bargaining. It is straightforward to interpret this condition. The parameter σ is a seller's share of the match value, and γ is the seller's marginal contribution to match formation. If the two are equal to each other, a seller is compensated by exactly his marginal contribution to match formation, in which case the different externalities from the seller's entry cancel out. Similarly, if $\sigma < \gamma$, a seller is under-compensated for his contribution, which leads to too few sites entering the market. If $\sigma > \gamma$, a seller is over-compensated for his contribution, which leads to too many sites entering the market.

4.2. Comparing efficiency between the two market mechanisms

Recall that the social welfare function is equal to a buyer's expected surplus in the market. Because both mechanisms induce only one quality level k to be produced in the equilibrium, the welfare function is equal to $W(k, \theta)$. There are three cases in which the comparison is simple. The first is $\sigma \leqslant \psi(1) + c$. In this case, price posting is clearly superior to bargaining, because the market is active under price posting but shuts down under bargaining. The second case is $\lambda \in [\underline{k}, \gamma]$ and $\sigma \neq \gamma$. In this case, the price-posting equilibrium is socially efficient (see Case H1 in Table 1), but the equilibrium under bargaining is not. Again, price posting is superior to bargaining. The third case is one where $\sigma = \gamma$ and $\lambda > \gamma$. This case is opposite to the second case. That is, bargaining yields the socially efficient outcome but price posting does not.

Consider the remaining case where $\lambda > \gamma$, $\sigma > \psi(1) + c$ and $\sigma \neq \gamma$. In this case, both mechanisms induce the market to be active and generate inefficiency. However, since the two mechanisms do not always induce the same quality to be produced in the equilibrium, we divide the comparison further into two cases:

Case H2: $\lambda \in (\gamma, \lambda_c)$. In this case, both mechanisms induce the high quality to be produced in the equilibrium. Welfare is equal to $W(1, \theta_1(\lambda))$ under price posting and $W(1, \theta_b(\sigma))$ under bargaining, where the notation $\theta_b(\sigma)$ emphasizes the dependence of θ_b on σ. In Appendix B, we prove that there exists a function $s(\sigma)$ such that bargaining yields higher welfare than price posting in this case if and only if $\lambda > s(\sigma)$. This is intuitive because, as λ increases further above γ, posting price at the inefficiently high level λ to signal high quality becomes increasingly more costly.

Case L: $\lambda \in (\lambda_c, 1)$. In this case, the low quality is produced under price posting, but the high quality is produced on bargaining. The welfare level is equal to $W(\lambda, \theta_\lambda(\lambda))$ under price posting and $W(1, \theta_b(\sigma))$ under bargaining. In Appendix B, we prove that there exists a function $r(\sigma)$ such that bargaining yields higher welfare than price posting in this case if and only if $\lambda < r(\sigma)$. To explain this condition, recall that price posting in this case generates the efficient division of the match surplus between buyers and sellers, given the low quality. All of the inefficiency under price posting, including the deficient number of sites, is caused by the low quality. As the low quality increases, this inefficiency falls, and so welfare increases, in contrast to Case H2. In the limit $\lambda \to 1$, price posting yields the efficient allocation. Thus, for bargaining to be superior to price posting in this case, the low quality cannot be too high, as described by the condition $\lambda < r(\sigma)$.

We put the cases together in the following proposition (see Appendix B for a proof):

Proposition 4.2. *Suppose that buyers observe the quality of the good in their match. Define the functions $s(\sigma)$ and $r(\sigma)$ as in Appendix B. Denote*

$$B = \{(\lambda, \sigma) \in [\underline{k}, 1] \times [0, 1]\colon\ s(\sigma) < \lambda < r(\sigma)\}. \tag{4.2}$$

Let B^c include all values of $(\lambda, \sigma) \in [\underline{k}, 1] \times [0, 1]$ that are not in B or on the boundary of B. Then, bargaining generates higher welfare than price posting if $(\lambda, \sigma) \in B$ and $\sigma > \psi(1) + c$, lower welfare than price positing if $(\lambda, \sigma) \in B^c$ or $\sigma \leqslant \psi(1) + c$, and the same welfare as price posting if the values of (λ, σ) are on the boundary of B and satisfy $\sigma > \psi(1) + c$. The subset of B that satisfies $\sigma > \psi(1) + c$ is a connected open subset of $[\underline{k}, 1] \times [0, 1]$ and has positive measure. Furthermore, $s(\sigma)$ and $r(\sigma)$ have the following properties: (i) *$s'(\sigma) < 0$ for $\sigma < \gamma$, and $s(\sigma) = \sigma$ for $\sigma \geqslant \gamma$*; (ii) *$r'(\sigma) > 0$ iff $\sigma < \gamma$, and the maximum of $r(\sigma)$ is $r(\gamma) = 1$*; (iii) *$r(\sigma) = s(\sigma)$ for all σ that satisfy $s(\sigma) = \lambda_c$.*

Fig. 1 depicts the set B as the shaded region, for the case where the entire set B satisfies $\sigma > \psi(1) + c$. In this case, bargaining generates higher welfare than price posting inside the set B, while price posting generates higher welfare outside the set B. Since the subset of B that satisfies $\sigma > \psi(1) + c$ has positive measure, there is a generic set of economies in which bargaining is more efficient than price posting. Similarly, since the complementary set of B has positive measure, there is also a generic set of economies in which price posting is more efficient than bargaining. In contrast, the set of economies in which the two pricing mechanisms generate the same welfare has zero measure.[18]

[18] If $\sigma_1 < \psi(1) + c$, where σ_1 is the smaller solution to $s(\sigma) = r(\sigma)$, then price posting also generates higher welfare than bargaining in the part of B that lies between $\sigma \geqslant \sigma_1$ and $\sigma \leqslant \psi(1) + c$. However, since $\gamma > \psi(1) + c$, the part of B that satisfies $\sigma > \psi(1) + c$ always has positive measure.

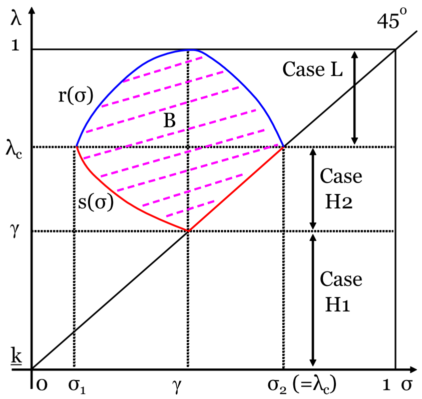

Fig. 1. Relative efficiency between bargaining and price positing.

To explain the relative efficiency between the two pricing mechanisms, note that price posting (with directed search) and bargaining generate potential inefficiency for different reasons. Under price positing, potential inefficiency is caused by the constraint on signaling quality, which can lead to inefficient entry of sites or the choice of inefficient quality. This potential inefficiency depends on the relative quality of the low-quality good, λ, but it does not depend on a seller's bargaining power σ. Under bargaining, the choice of quality is always efficient, but the division of the match surplus between buyers and sellers generically fails to internalize matching externalities, in which case the entry of sites is inefficient. This potential inefficiency depends on a seller's bargaining power, σ. Thus, when these two parameters take on values close to their bounds, the comparison between the two mechanisms are trivial. For example, if $\lambda \leqslant \gamma$, the equilibrium under price posting is socially efficient and, hence, more efficient than bargaining as long as $\sigma \neq \gamma$. Also, price posting is more efficient than bargaining if the inefficiency under bargaining is high as a result of either too little entry of sites, which occurs if $\sigma \leqslant \max\{\sigma_1, \psi(1) + c\}$, or too much entry of sites, which occurs if $\sigma \geqslant \sigma_2$.

Next, let us fix a seller's bargaining power at a value in $(\max\{\sigma_1, \psi(1) + c\}, \sigma_2)$, and vary the relative quality of the low-quality good from γ to 1. As the relative quality λ increases from γ, the gap between the price that signals quality, λ, and the price that efficiently directs search, γ, increases. Thus, the inefficiency under price posting increases. When λ increases above the threshold $s(\sigma)$, this inefficiency under price posting becomes sufficiently large that price posting is less efficient than bargaining. As λ increases further above the level λ_c, a seller finds it optimal to produce the low quality rather than the high quality under price posting. If the low quality is still much lower than the high quality, the inefficiency from the low quality is sufficiently high that price posting is less efficient than bargaining. Because sellers have no need to signal quality when they produce the low quality, the inefficiency under price posting falls as the relative quality of the low-quality good rises further toward one. When λ goes above the threshold $r(\sigma)$, the inefficiency caused by the low quality is sufficiently small that price posting becomes more efficient than bargaining. Furthermore, the interval of λ in which bargaining is more efficient than price posting depends on the value of σ. Not surprisingly, the closer is σ to the efficient level γ, the wider is the interval of λ in which bargaining dominates price posting.

Notice that we can use the above analysis to answer the question of which mechanism – price posting or bargaining – would survive in an equilibrium where agents can choose between a "price posting island" (with many submarkets) and a "bargaining island". Assume that buyers choose between the two islands after observing prices on the posting island. Also, assume that the only reason for an island to be inactive in equilibrium is that it generates a lower expected surplus for buyers than the other island does. As competitive entry of sellers drives down a seller's expected profit to zero, buyers' expected surplus on an island is equal to the expected joint surplus on that island; i.e., $D = W(k, \theta)$ on each island. Only the island with the higher social welfare level will be active in equilibrium. Thus, Proposition 4.2 also tells us which mechanism would survive in such an environment.

Let us compare our results here with the literature. As mentioned earlier, Acemoglu and Shimer [1] study a directed search model with public information and find that price posting is superior to bargaining for almost all parameter values. Bester [3] incorporates private information but abstracts from matching entirely. Instead, he models search cost as time discounting, as a buyer must take one period to opt out from the described market to an outside good which has a uniform quality. To compare his results with ours, it is best to set the discount factor to zero because our model has one period and the payoff to an individual from not trading is zero. In this case, Bester's model predicts that bargaining is superior to price posting if and only if sellers' bargaining power is small, while our model predicts the opposite when sellers' bargaining power is small. An important cause of this difference is that matching frictions do not exist in Bester's model when the discount rate goes to zero, and so sellers' small bargaining power does not cause inefficient entry of selling sites as in our model. Finally, Michelacci and Suarez [17] compare price posting with bargaining in a directed search model of the labor market with adverse selection. They show that the two pricing mechanisms can coexist in a positively measured parameter region and, in this region, wage-posting generates higher welfare than bargaining. The main reason for this difference between their result and ours is that our model involves signaling while their model involves adverse selection. Another reason is that, in our model, the total supply and the composition of sites whose information is private are determined endogenously by competitive entry. In their model, these dimensions are fixed and, instead, competitive entry occurs on the side of the market that does not have private information.

5. Discussion

In this section, we discuss the following assumptions of the model: (i) a buyer receives an exogenous signal about the quality (in addition to the price signal) after being matched with a site; (ii) a seller chooses the quality of the good; (iii) a seller posts a single price; and (iv) sellers are the individuals who incur the cost to create trading sites. The discussions on (i) and (ii) can help relating and contrasting to a standard signaling model such as MR, while the discussions on (iii) and (iv) can help understanding further the potential conflict between the two roles of a posted price.

5.1. Information acquisition

We have assumed that a buyer receives a free signal about the quality of the good after being matched with a site, in addition to the price signal. This raises the question about information acquisition. If the signal costs $\varepsilon > 0$ instead, will a buyer choose to incur the cost after being matched? The answer is no, no matter how small the cost is. Because prices in the separating equilibrium have already revealed all relevant information about the quality, additional signals about the quality have no value to the buyer. But if no buyer acquires the information, then it is difficult to support a separating equilibrium; specifically, Restriction 1 and some parts of

Lemma 3.2 are no longer valid. This standard paradox of information acquisition, highlighted by Grossman and Stiglitz [10], arises because the information contained in prices is a public good.

However, information acquisition can take place if we modify the model as follows. Suppose that each buyer can choose to participate in one of two groups of submarkets. The first group of submarkets are just like the ones in the current model, where a buyer receives the signal about the quality of the good after being matched with a site. A buyer must incur a cost $\varepsilon > 0$ in order to participate in these submarkets. In the other group of submarkets, participation is free, but a buyer does not receive a signal in addition to the price. Participation in the first group of submarkets can be interpreted as information acquisition. When ε is sufficiently small, it is possible that participating in the first group of submarkets yields a higher expected surplus to a buyer than participating in the second group of submarkets, because the latter may result in pooling and a lower average quality. In this case, information acquisition occurs in the equilibrium. However, working out this extension of the model is beyond the scope of this paper.

5.2. Exogenous quality and pooling equilibria

We have assumed that sellers can choose the quality of the good to be supplied at each site. Endogenous quality is important for the existence of the separating equilibrium in this model, because it justifies Restriction 2 on the beliefs associated with the plays out of equilibrium. To illustrate this importance, we consider the alternative environment where the distribution of the quality is fixed, as assumed in some well-known models of signaling, e.g. MR. In this alternative environment, we illustrate that there exists a continuum of pooling equilibria, that is, equilibria in which both quality levels are present in the same submarket. To simplify the exposition, we assume $\alpha = 1$ in this subsection.

Consider an economy where the mass of quality-k sellers is fixed at $\omega_k \in (0, 1)$, with $\omega_\lambda + \omega_1 = 1$. A type-$k$ seller is only able to produce the type-k good. Let S_k be the set containing all type-k sellers. We assume that each seller can produce at most one unit, but that he may choose to remain inactive. When entering a submarket, a quality-k seller must incur both the cost of the site and the cost of producing a quality-k good. The entry decision of a seller s is modified as $m_s(p)$, where $m_s(\cdot) : [0, 1] \to \{0, 1\}$. That is, the seller enters submarket p if $m_s(p) = 1$, and does not enter submarket p if $m_s(p) = 0$. The buyers observe only the value of $m_s(p)$ chosen by each seller s for all $p \in [0, 1]$, but they do not directly observe the seller's type. The aggregate collection of messages across sellers is given by $M = \{m_s\colon\ s \in S_\lambda \cup S_1\}$, and the total number of sites of quality-k goods in submarket p is $N(p, k) = \sum_{s \in S_k} m_s(p)$. Appendix C modifies Definition 3.1 of the equilibrium for this economy.

With exogenous quality, it is still true that a high-quality seller will not enter a submarket $p < \psi(1) + c$, because the price does not cover the cost. Also, it is not optimal for a low-quality seller to enter a submarket $p \geqslant \lambda$: under the assumption $\alpha = 1$, a price higher than the quality will result in no trade as the buyer will discover the quality of the good after being matched. So, parts (b) and (c) of Lemma 3.2 still hold. In addition, Restriction 1 on beliefs can be justified in the same way as before.

A main change is to part (a) of Lemma 3.2. With exogenous quality, part (a) of Lemma 3.2 holds for $p < \psi(1) + c$ but not for $p < \lambda$. It can be optimal for both types of sellers to enter a submarket where the price is below λ. To explain this change, recall that a seller can earn higher expected profit from entering submarket $p < \lambda$ with a low-quality good than with a high-quality good, regardless of buyers' beliefs. When the quality is a seller's choice, this profit differential induces all entries into a submarket $p < \lambda$ to be with low-quality goods instead of high-quality goods. When the quality choice is not available, however, different types of sellers can earn different profits in the market, and hence the two types can be present in the same submarket $p < \lambda$ if $\lambda > \psi(1) + c$.

This change to Lemma 3.2 implies that Restriction 2 can no longer be justified. Consequently, pooling equilibria can exist. It should be clear from the above analysis that pooling of the two qualities occurs only in submarkets $p \in (\psi(1) + c, \lambda)$. Assuming $\lambda > \psi(1) + c$, Appendix C proves that there is a non-empty parameter region in which pooling in submarket $p_p \in (\psi(1) + c, \lambda)$ is an equilibrium. This pooling equilibrium has submarket p_p as the only active submarket and is supported by buyers' beliefs that $\mu(p_p) = \omega_H$, $\mu(p) = 0$ for all $p < \lambda$ with $p \neq p_p$, and $\mu(p) = 1$ for $p \geqslant \lambda$. Moreover, there is an interval of $\hat{p}$ near p_p each of which is a pooling equilibrium, supported by buyers' beliefs that $\mu(\hat{p}) = \omega_H$, $\mu(p) = 0$ for all $p < \lambda$ with $p \neq \hat{p}$, and $\mu(p) = 1$ for $p \geqslant \lambda$. Because pooling occurs only at prices below λ, its existence relies on the search-directing role of a posted price. If there were no need to direct search, a high-quality seller would set the price above λ to separate from a low-quality seller.

5.3. More elaborate pricing mechanisms

In the analysis so far, a submarket is described by a single price that a buyer pays in a trade. One can consider a more general setup where each submarket is described by a pair of prices, $p = (p_B, p_A)$, where the subscript B means "before" and A "after". One example of this two-part pricing scheme involves "money burning" by sellers when they enter a submarket. In this scheme, $-p_B > 0$ is the amount that a seller spends (burns) on each site in addition to the cost c before matching takes place, and p_A is the price that a buyer will pay for the good. Because the amount $-p_B$ is observable by buyers, it may help a high-quality site signaling quality. Although the addition of p_B to the description of a submarket increases the capacity for a seller to signal quality, such spending is a waste to the society. Thus, any equilibrium with $-p_B > 0$ is socially inefficient.

Another example of the two-part pricing scheme involves a transfer between a buyer and a seller before the buyer receives a signal about the quality of the good. Specifically, p_B is the amount that a buyer pays a seller immediately after the two are matched but before the buyer receives a signal. After receiving a signal, if the buyer chooses to buy the good, the buyer pays the additional amount p_A. The amount p_B need not be positive. If $p_B > 0$, it is a fee that a buyer pays in order to inspect the good; if $p_B < 0$, it is a door prize received by the buyer as a reward for the match. In this environment, there are equilibria that implement the socially efficient allocation for all values of $\lambda < 1$, not just for $\lambda \leqslant \gamma$ as in Proposition 3.3. One of these equilibria is as follows. All sellers produce high-quality goods and enter submarket $p^* = (p_B^*, p_A^*)$, where $p_B^* = \gamma - 1 + \varepsilon$, $p_A^* = 1 - \varepsilon$, and $\varepsilon > 0$ is a sufficiently small number. Notice that $p_B^* < 0$, $p_A^* > \lambda$ (provided $\lambda < 1 - \varepsilon$) and $p_B^* + p_A^* = \gamma$. Suppose for the moment that there is a neighborhood of p^* in which $\mu(\tilde{p}) = 1$ for all $\tilde{p}$ in the neighborhood. Then, the expected profit to a (high-quality) site and a buyer's expected surplus in submarket $\tilde{p}$ are, respectively,

$$\pi(\tilde{p}, 1) = (\tilde{p}_B + \tilde{p}_A)F\big(\theta(\tilde{p})\big)/\theta(\tilde{p}) - \psi(1) - c, \qquad D(\tilde{p}) = F\big(\theta(\tilde{p})\big)(1 - \tilde{p}_B - \tilde{p}_A).$$

The solution to the problem of maximizing $D(p)$ subject to $\pi(p, 1) = 0$ yields $p_B + p_A = \gamma$. Thus, p^* is a solution to such a maximization problem. It is easy to see that this solution implements the socially efficient allocation. Now, we can justify the beliefs $\mu(\tilde{p}) = 1$ for $\tilde{p}$ in a small neighborhood of p^*. Consider a seller who enters submarket $\tilde{p}$ in this neighborhood with a low-quality good. Consider the belief that is most favorable to the seller, i.e., that buyers believe all sites in $\tilde{p}$ to have high quality. Even with this favorable belief, a low-quality site's expected revenue in submarket $\tilde{p}$ is $(1-\alpha)\tilde{p}_A + \tilde{p}_B$ (as $\tilde{p}_A > \lambda$). Because $\tilde{p}_B < 0$, $\tilde{p}_A < 1$ and $\alpha > \alpha_0$, this expected revenue is strictly less than $1 - \alpha_0 = \psi(\lambda) + c$ and, hence, expected profit of the site is strictly negative. Thus, similar to Restriction 1, we can justify the restriction on the beliefs that $\mu(\tilde{p}) = 1$ for $\tilde{p}$ sufficiently close to p^*.

Given the result that this two-part pricing scheme always implements the socially efficient allocation, it is interesting to ask why the scheme is not used more often in reality than the single-price scheme. One possible explanation is that it may require a seller to give a buyer a positive transfer before any trade occurs. In the above example, this transfer is $-p_B^* = 1 - \gamma - \varepsilon > 0$. More generally, all equilibria with the above two-part pricing scheme that induce efficient entry of sites must have $p_B + p_A = \gamma$ in order to internalize search externalities, and all equilibria that enable sellers to signal high quality must have $p_A \geqslant \lambda$ to prevent low-quality sellers from mimicking. Thus, a necessary condition for efficiency is $-p_B \geqslant \lambda - \gamma$. This is positive when $\lambda > \gamma$. That is, whenever the single-price scheme generates inefficiency, the two-part pricing scheme can improve efficiency only if a seller makes a positive transfer to a buyer before trade occurs. This may not always be feasible if sellers are financially constrained. Another reason why the two-part pricing scheme is less commonly observed than the single-price scheme is that it may be more difficult to commit to a two-part pricing scheme than to a single-price scheme.

5.4. *Adverse selection: buyers creating sites*

Now let us discuss the alternative environment where buyers, rather than sellers, create trading sites. Precisely, buyers choose which submarket to enter and how many buying sites to create in a submarket before sellers choose which submarket to visit. For each site, a buyer must incur the cost c before the site can be in the matching process in the submarket. After observing the distribution of sites across the submarkets, each seller chooses which submarket to enter and the quality of the good to produce. Because buyers enter the market first to create sites, buyers are the ones who "post" prices to direct search in this environment. The resulting information problem is adverse selection, rather than signaling. One can define an equilibrium similarly to Definition 3.1 by switching the roles of buyers and sellers. However, there is no counterpart to Restrictions 1 and 2 that restrict the beliefs on out-of-equilibrium plays, because the individuals who enter the market first (i.e., buyers) do not have private information. As in Guerrieri et al. [12], an equilibrium features separation between high- and low-quality; i.e., sellers with different qualities self-select into submarkets that differ in posted prices and the associated tightness. Generically, only one type of good is produced in the equilibrium, as in the signaling equilibrium.

The main difference between the equilibrium with adverse selection and the equilibrium with signaling is whether the market is always active in the equilibrium. Under Assumptions 1 and 2, the market is always active in the equilibrium with signaling (see Proposition 3.3). Under the same assumptions, however, the market may shut down in the equilibrium with adverse selection. This contrast between the two environments is intuitive. With signaling, because a seller incurs both the cost of producing a good and the cost of setting up a site for the good, the seller will choose to enter a submarket that can compensate for both costs. And there are submarkets that can do so. For example, if the price in a submarket is only slightly higher than the sum of the two costs, very few sites are expected to enter the submarket but many buyers are expected to enter. In this case, the matching probability for a site in the submarket is close to one, and so the expected revenue of a site will be able to cover the sum of the costs. This is not necessarily the outcome in the environment with adverse selection. In particular, if the cost of a site is sufficiently high, the number of buying sites in a submarket is small. The matching probability for a seller in the submarket can be so low that a seller's expected revenue is strictly lower than the cost of producing a good. In this case, the market shuts down.

This contrast in the existence of an active market between the two environments arises even under public information. Moreover, in the parameter region where the two environments both have an active market, they may yield different welfare levels. These differences raise the general question of which side of the market should organize the market. We address this general question and provide the details for the above argument in a separate paper (Delacroix and Shi [7]).

Appendix A. Proofs for Section 3

A.1. *Proof of Lemma 3.2*

We prove Lemma 3.2 by examining the cases (a)–(c) below:

(a) $p < \psi(1) + c$ or $p < \lambda$. If $p < \psi(1) + c$, a high-quality site in submarket p makes strictly negative profit even if the site is matched with a buyer with probability one and the buyer always buys the good. If $p < \lambda$, then $\phi(\mu) \geqslant \lambda > p$ for all μ. In this case, a buyer in a match in submarket p will always buy the good regardless of the signal. Then, (3.2) shows that $\pi_\mu(p, \lambda) - \pi_\mu(p, 1) = \psi(1) - \psi(\lambda) > 0$. It is not optimal for high-quality sites to enter submarket p.

(b) $p \geqslant \lambda$. Consider a low-quality site in submarket p. If $p = 1$, a buyer will not buy the good at all, and the expected profit of the site is strictly negative. If $p < 1$, a buyer will not buy a low-quality good when the signal is "true". Thus, the site's expected revenue from the match is at most equal to $(1 - \alpha)p$. Because $\alpha > \alpha_0 = 1 - \psi(\lambda) - c$ by Assumption 1, and because $p < 1$, this expected revenue is strictly less than $\psi(\lambda) + c$; i.e., expected profit from the low-quality site is strictly negative.

(c) $\lambda < \psi(1) + c$ and $\lambda \leqslant p < \psi(1) + c$. Since $p < \psi(1) + c$, the proof in (a) above implies that high-quality sites do not enter such a submarket p. Since $p \geqslant \lambda$, the proof in (b) above implies that low-quality sites do not enter such a submarket p. □

A.2. *Multiplicity of equilibria without Restrictions 1 and 2*

We now establish multiplicity of equilibria as defined in Definition 3.1. As discussed in Section 3.2, consider the singleton set $P = \{p_a\}$, where $\max\{\lambda, \psi(1) + c\} \leqslant p_a < 1$, together with the beliefs $\mu(p_a) = 1$ and $\mu(p) = 0$ for all $p \neq p_a$. Define θ_a as the solution to: $p_a F(\theta_a)/\theta_a = \psi(1) + c$. Denote $D_a = F(\theta_a)(1 - p_a)$ and, for $p < \lambda$, denote $\Theta_a = F^{-1}(D_a/(\lambda - p))$. Then, (p_a, θ_a, D_a) and the specified beliefs together constitute an equilibrium as in Definition 3.1 if the following condition is met:

$$\text{either } D_a > \lambda - p \quad \text{or } pF(\Theta_a)/\Theta_a < \psi(\lambda) + c, \quad \text{all } p < \lambda.$$

It can be verified that, for sufficiently small λ, there is a range of values of p_a that satisfy the above condition. In this case, there is a continuum of equilibria.

To see why the above construction yields an equilibrium, note first that θ_a is well-defined and $D_a > 0$, because $p_a \geqslant \psi(1) + c$ and $p_a < 1$. Also, because $p_a \geqslant \lambda$, part (a) of Lemma 3.2 implies that it is optimal for a seller who enters submarket p_a to produce the high-quality good. Since $P = \{p_a\}$ by construction, requirements (iii)–(iv) in Definition 3.1 are trivially satisfied. Because the construction of θ_a and the specified beliefs ensure that expected profit of a site in submarket p_a is zero, requirement (i) is satisfied for $p = p_a$. Requirement (ii) is also satisfied for $p = p_a$ if visiting submarkets other than p_a does not give a buyer higher expected utility than D_a. Thus, it suffices to verify that requirements (i)–(ii) in Definition 3.1 are satisfied for all $p \neq p_a$. For $p \geqslant \max\{\lambda, \psi(1) + c\}$ and $p \neq p_a$, the specified belief $\mu(p) = 0$ implies $\phi(\mu) = \lambda < p$, and so (3.1) yields $D_\mu(p) = 0 < D_a$. The complementary slackness condition in requirement (ii) implies $\theta_\mu(p) = +\infty$, and (3.2) yields $\pi_\mu(p, k) = -\psi(k) - c < 0$ for $k \in \{\lambda, 1\}$. For $\lambda \leqslant p < \max\{\lambda, \psi(1) + c\}$, part (c) of Lemma 3.2 implies that it is never optimal to enter such a submarket. For $p < \lambda$, part (a) of Lemma 3.2 implies that we only need to consider a low-quality site. Suppose that such a site attract buyers. Then, the expected surplus to a buyer must be at least D_a; that is, $F(\theta_\mu(p)) \geqslant D_a/(\lambda - p)$, where we have used the fact that a buyer in a match in the submarket will always buy the good. If $D_a > \lambda - p$, this condition cannot be satisfied. If $D_a \leqslant \lambda - p$, the condition is equivalent to $\theta_\mu(p) \geqslant \Theta_a$, where Θ_a is defined above. In this case, expected profit of a low-quality site in submarket $p < \lambda$ is strictly negative if $pF(\Theta_a)/\Theta_a < \psi(\lambda) + c$, as specified above. □

A.3. Derivation of (3.9)

We derive (3.9) from the maximization problem in (3.8). From the constraint $\pi(p, 1) = 0$ we can solve $p = [\psi(1) + c]\theta/F(\theta)$. This relationship between p and θ is exhibited in (3.9) for submarket p_1. Substituting p with this relationship, we can change the choice variable from p to θ. The constraint $p \geqslant p_0$ becomes $F(\theta)/\theta \leqslant [\psi(1) + c]/p_0$, which is also exhibited in (3.9) for submarket p_1. The objective function in the maximization problem becomes: $f(\theta) = F(\theta) - \theta[\psi(1) + c]$. Under Assumption 2, $f(\theta)$ is strictly concave and, hence, has a single peak. The peak occurs at $\theta^*_H = \theta^*(1)$, where $\theta^*(k)$ is defined by (2.2). If $F(\theta^*_H)/\theta^*_H \leqslant [\psi(1)+c]/p_0$, then the implied price p_1 satisfies the constraint $p_1 \geqslant p_0$, and so $\theta_1 = \theta^*_H$. In this case, substituting $\psi(1) + c = F'(\theta^*_H)$ yields $p_1 = p^*_H = G(\theta^*_H)$, where G is defined in (2.3). If $F(\theta^*_H)/\theta^*_H > [\psi(1) + c]/p_0$, then the optimal choice, θ_1, satisfies $F(\theta_1)/\theta_1 = [\psi(1) + c]/p_0 < F(\theta^*_H)/\theta^*_H$. Because $F(\theta)/\theta$ is a strictly decreasing function, then $\theta_1 > \theta^*_H$ in this case. In both cases, the two inequalities in (3.9) hold with complementary slackness. The maximized value of the objective function is $D_1 = f(\theta_1) = F(\theta_1) - \theta_1[\psi(1) + c]$, as exhibited in (3.9).

A.4. Proof of Proposition 3.3

Let us first analyze how (p_1, θ_1, D_1) and $(p_\lambda, \theta_\lambda, D_\lambda)$ depend on λ. Start with (p_1, θ_1, D_1), which are given in (3.9). Let us write p_0 defined in Restriction 1 as $p_0(\lambda)$. If $\theta_1 = \theta^*_H$, then the complementary slackness condition in (3.9) requires: $p_0(\lambda) \leqslant \frac{\theta^*_H[\psi(1)+c]}{F(\theta^*_H)} = \gamma$, where the equality follows from the fact that $\psi(1) + c = F'(\theta^*_H)$ and γ is defined in Proposition 3.3. If $\theta_1 > \theta^*_H$, the complementary slackness condition in (3.9) requires

$$p_0(\lambda) = \frac{\theta_1[\psi(1) + c]}{F(\theta_1)} > \frac{\theta^*_H[\psi(1) + c]}{F(\theta^*_H)} = \gamma,$$

where the inequality follows from the assumption that F/θ is a strictly decreasing function. Recall that $\gamma > \psi(1) + c$. If $\lambda \leqslant \gamma$, then $p_0(\lambda) \leqslant \gamma$. If $\lambda > \gamma$, then $p_0(\lambda) = \lambda > \gamma$. That is, $p_0(\lambda) > \gamma$ if and only if $\lambda > \gamma$. Thus, we can write θ_1 as a function of λ:

$$\theta_1(\lambda) = \begin{cases} \theta^*_H, & \text{if } \lambda \leqslant \gamma, \\ \theta_1 \text{ such that } \frac{F(\theta_1)}{\theta_1} \leqslant \frac{1}{\lambda}[\psi(1) + c], & \text{if } \lambda > \gamma. \end{cases}$$

Similarly, we can write p_1 and D_1 as

$$p_1(\lambda) = \frac{[\psi(1)+c]\theta_1(\lambda)}{F(\theta_1(\lambda))},$$
$$D_1(\lambda) = F\big(\theta_1(\lambda)\big) - \theta_1(\lambda)\big[\psi(1)+c\big].$$

If $\lambda \leqslant \gamma$, then $\psi(1)+c = F'(\theta_1)$ and $\theta_1 = \theta_H^*$, which imply then $p_1(\lambda) = \gamma$. If $\lambda > \gamma$, then $p_1(\lambda) = p_0(\lambda) = \lambda$. These two cases of (p_1, θ_1) are listed as Case H1 and Case L in Table 1.

Note that $\theta_1'(\lambda) = 0$ for $\lambda \leqslant \gamma$, and $\theta_1'(\lambda) > 0$ for $\lambda > \gamma$. Thus, $p_1'(\lambda) = 0$ for $\lambda \leqslant \gamma$, and $p_1'(\lambda) = 1$ for $\lambda > \gamma$. Similarly, if $\lambda \leqslant \gamma$, then $D_1'(\lambda) = 0$; if $\lambda > \gamma$, then $D_1'(\lambda) < 0$ because $F'(\theta_1) < F'(\theta_H^*) = \psi(1) + c$ in this case. Moreover, as $\lambda \nearrow 1$, $[\frac{F(\theta_1(\lambda))}{\theta_1(\lambda)}] \searrow [\psi(1)+c]$, and so $D_1(\lambda) \searrow 0$. Thus, $D_1(\lambda) > 0$ for all $\lambda < 1$.

Now we analyze $(p_\lambda, \theta_\lambda, D_\lambda)$, which are given in (3.11). The pair $(p_\lambda, \theta_\lambda)$ is listed in Case L in Table 1. Since $\theta_\lambda = \theta^*(\lambda)$ and $\theta^*(\cdot)$ (defined in (2.2)) is strictly increasing, then $\frac{d\theta_\lambda}{d\lambda} > 0$. This implies that $\theta_\lambda < \theta^*(1) = \theta_H^*$ for all $\lambda < 1$. Also, we can substitute $F'(\theta_\lambda) = [\psi(\lambda)+c]/\lambda$ to rewrite $p_\lambda = \frac{\theta_\lambda[\psi(\lambda)+c]}{F(\theta_\lambda)}$. Because $\frac{\theta}{F(\theta)}$ is strictly increasing in θ and $\psi(\lambda)$ is strictly increasing in λ, the result $\frac{d\theta_\lambda}{d\lambda} > 0$ implies that $\frac{dp_\lambda}{d\lambda} > 0$. Moreover, $p_\lambda < \frac{\theta_H^*}{F(\theta_H^*)}[\psi(1)+c] = \gamma$ for all $\lambda < 1$. For the dependence of D_λ on λ, rewrite $D_\lambda = F(\theta_\lambda)\lambda - \theta_\lambda[\psi(\lambda)+c]$. We have

$$\frac{dD_\lambda}{d\lambda} = F(\theta_\lambda) - \theta_\lambda\psi'(\lambda) > F(\theta_\lambda) - \theta_\lambda\frac{\psi(\lambda)+c}{\lambda} = \frac{D_\lambda}{\lambda}.$$

The first equality comes from the envelope condition. The strict inequality comes from the assumptions $\psi'(\lambda) \leqslant \psi(\lambda)/\lambda$ and $c > 0$. The last equality comes from the expression for D_λ. Thus, $D_\lambda \geqslant 0$ implies $dD_\lambda/d\lambda > 0$. When $\lambda = \underline{k}$, we have $F'(\theta_\lambda) = 1$, in which case $\theta_\lambda = 0$ and $D_\lambda = 0$. Thus, $dD_\lambda/d\lambda > 0$ at $\lambda = \underline{k}$. For sufficiently small $\varepsilon > 0$, we have $D_\lambda(\underline{k}+\varepsilon) > D_\lambda(\underline{k}) = 0$ which, in turn, implies that $dD_\lambda/d\lambda > 0$ at $\lambda = \underline{k}+\varepsilon$. Induction yields $dD_\lambda/d\lambda > 0$ and $D_\lambda > 0$ for all $\lambda \in (\underline{k}, 1)$. Moreover, when $\lambda \nearrow 1$, we have $\theta_\lambda \nearrow \theta_H^*$ and $D_\lambda \nearrow D_1(\gamma)$.

Because $D_1(\lambda) > 0$ for all $\lambda < 1$ and $D_\lambda(\lambda) > 0$ for all $\lambda > \underline{k}$, then $\max\{D_1(\lambda), D_\lambda(\lambda)\} > 0$ for all $\lambda \in [\underline{k}, 1]$. That is, the market is always active, as stated in (i) of Proposition 3.3. Moreover, from the above proof, we know that (a) $D_1(\lambda)$ is decreasing, and strictly so for $\lambda > \gamma$; (b) $D_\lambda(\lambda)$ is strictly increasing; (c) $D_\lambda(\underline{k}) = 0 < D_1(\underline{k})$ and $D_\lambda(1) = D_1(\gamma) > 0 = D_1(1)$. These results imply that there exists a unique $\lambda_c \in (\gamma, 1)$ such that $D_1(\lambda_c) = D_\lambda(\lambda_c)$. The equilibrium is unique for all $\lambda \neq \lambda_c$. If $\lambda < \lambda_c$, then $D_1 > D_\lambda$, in which case only submarket p_1 (with high-quality goods) is active in the equilibrium. If $\lambda > \lambda_c$, then $D_1 < D_\lambda$, in which case only submarket p_λ (with low-quality goods) is active in the equilibrium. Case H1 in Table 1 corresponds to the high-quality equilibrium where the constraint $p_1 \geqslant \lambda$ does not bind, Case H2 to the high equilibrium where the constraint $p_1 \geqslant \lambda$ binds, and Case L to the low-quality equilibrium. The above proof has already shown that the price and the tightness in these cases are those listed in Table 1. This establishes (ii) of Proposition 3.3.

Part (iii) of Proposition 3.3 is evident because, in Case H1, the high-quality equilibrium yields the same quality and tightness as the socially efficient allocation does. Part (iv) of the proposition also follows from the above analysis on the price and the tightness in the equilibrium. Finally, in the text preceding Proposition 3.3, we have explained that welfare in the equilibrium is $D = W(k, \theta)$, where k is the quality, and θ the tightness, in the active submarket. This result is shown as the last column in Table 1. □

Appendix B. Proof of Proposition 4.2

The analysis in Section 4.2 preceding Proposition 4.2 has already compared the two pricing mechanisms in the case where $\sigma \leqslant \psi(1)+c$, the case where $\lambda \in [\underline{k}, \gamma]$ and $\sigma \neq \gamma$, and the case where $\sigma = \gamma$ and $\lambda > \gamma$. For the remaining parameter region, where $\lambda > \gamma$, $\sigma > \psi(1)+c$ and $\sigma \neq \gamma$, the analysis has also listed the two cases, Case H2 and Case L. For these two cases, we compare the two pricing mechanisms in more details and establish properties (i)–(iii) in Proposition 4.2.

Case H2: $\lambda \in (\gamma, \lambda_c)$. In this case, welfare is higher under bargaining than under price posting if and only if $W(1, \theta_1(\lambda)) < W(1, \theta_b(\sigma))$, where $\theta_b(\sigma)$ is defined by (4.1) and $\theta_1(\lambda)$ by (3.9). Using the expression for W in (2.1), we find that $W(1,\theta)$ is increasing in θ if and only if $F'(\theta) > \psi(1)+c$, i.e., if and only if $\theta < \theta_H^*$. Because $\theta_1(\lambda) > \theta_H^*$ (as $\lambda > \gamma$), and because $\theta_1(\lambda)$ is an increasing function (see the proof of Proposition 3.3), then $W(1, \theta_1(\lambda))$ is decreasing in λ. Let us define $s(\sigma)$ by

$$W\bigl(1, \theta_1\bigl(s(\sigma)\bigr)\bigr) = W\bigl(1, \theta_b(\sigma)\bigr).$$

Then, $s(\sigma)$ exists and is unique for each σ. Moreover, $W(1, \theta_1(\lambda)) < W(1, \theta_b(\sigma))$ if and only if $\lambda > s(\sigma)$.

By the definition of θ_H^* in (2.4), $W(1,\theta)$ is increasing in θ if and only if $\theta < \theta_H^*$. Because $\theta_b(\sigma) < \theta_H^*$ if and only if $\sigma < \gamma$ (see Proposition 4.1), then $W(1, \theta_b(\sigma))$ is increasing in σ if and only if $\sigma < \gamma$. The definition of $s(\sigma)$ then implies that $s(\sigma)$ is a decreasing function if and only if $\sigma < \gamma$, as listed in (i) in Proposition 4.2. Because $\theta_b(\gamma) = \theta_H^*$, then $s(\sigma) = \gamma$. Consider the case where $\sigma > \gamma$. In this case, $\theta_b(\sigma) > \theta_H^*$. Since $\theta_1(\lambda) > \theta_H^*$, and since $W(1,\theta_1)$ and $W(1,\theta_b)$ are both decreasing in θ in this case, then $W(1,\theta_1) < W(1,\theta_b)$ if and only if $\theta_1 > \theta_b$. Recall that $F(\theta_1)/\theta_1 = [\psi(1)+c]/\lambda$, and $F(\theta_b)/\theta_b = [\psi(1)+c]/\sigma$. Because $F(\theta)/\theta$ is a decreasing function, then $\theta_1 > \theta_b$ if and only if $\lambda > \sigma$ in the case $\sigma > \gamma$. That is, $s(\sigma) = \sigma$ in the case $\sigma > \gamma$, as listed in (i) in Proposition 4.2.

Case L: $\lambda \in (\gamma, \lambda_c)$. In this case, welfare is higher under bargaining than under price posting if and only if $W(\lambda, \theta_\lambda(\lambda)) < W(1, \theta_b(\sigma))$. Because $\frac{d}{d\lambda} W(\lambda, \theta_\lambda(\lambda)) > 0$ (see the proof of Proposition 3.3), then we can define a unique function $r(\sigma)$ by

$$W\bigl(r(\sigma), \theta_\lambda\bigl(r(\sigma)\bigr)\bigr) = W\bigl(1, \theta_b(\sigma)\bigr).$$

Moreover, $W(\lambda, \theta_\lambda(\lambda)) < W(1, \theta_b(\sigma))$ if and only if $\lambda < r(\sigma)$. As proven in Case H2 above, $W(1, \theta_b(\sigma))$ is increasing in σ if and only if $\sigma < \gamma$. The above definition of $r(\sigma)$ then implies that $r(\sigma)$ is an increasing function if and only if $\sigma < \gamma$, as listed in (ii) in Proposition 4.2. Thus, $r(\sigma)$ is maximized at $\sigma = \gamma$. At $\sigma = \gamma$, we have $\theta_b(\gamma) = \theta_H^*$. Since $W(\lambda, \theta_\lambda(\lambda)) \to W(1, \theta_H^*)$ as $\lambda \to 1$ (see the proof of Proposition 3.3), then $r(\gamma) = 1$, as listed in (ii) in Proposition 4.2.

To prove (iii) in Proposition 4.2, note that the equation $s(\sigma) = \lambda_c$ has two solutions: one is $\sigma_1 < \gamma$ and the other is $\sigma_2 = \lambda_c > \gamma$. For both $i = 1, 2$, the definition of $s(\sigma)$ implies

$$W\bigl(1, \theta_b(\sigma_i)\bigr) = W\bigl(1, \theta_1\bigl(s(\sigma_i)\bigr)\bigr) = W\bigl(1, \theta_1(\lambda_c)\bigr).$$

By the definition of λ_c, $W(1, \theta_1(\lambda_c)) = W(\lambda_c, \theta_\lambda(\lambda_c))$. Thus, $W(\lambda_c, \theta_\lambda(\lambda_c)) = W(1, \theta_b(\sigma_i))$, $i = 1, 2$. By the definition of $r(\sigma)$, this result implies $r(\sigma_i) = \lambda_c$ for $i = 1, 2$. That is, $r(\sigma) = s(\sigma)$ if $s(\sigma) = \lambda_c$.

The above analysis implies that bargaining generates higher welfare than price posting if $(\lambda, \sigma) \in B$ and $\sigma > \psi(1)+c$, lower welfare than price positing if $(\lambda, \sigma) \in B^c$ or $\sigma \leqslant \psi(1)+c$, and the same welfare as price posting if the values of (λ, σ) are on the boundary of B that satisfies $\sigma > \psi(1) + c$. By the definition (4.2), the set B is open. B is connected because of properties (i)–(iii) in Proposition 4.2. Similarly, the subset of B that satisfies $\sigma > \psi(1) + c$ is open and connected. To see that this subset has positive measure, note that $s(\lambda_c) = \lambda_c > \gamma$, $s(\gamma) = \gamma < 1 = r(\gamma)$, and $\gamma > \psi(1) + c$. Thus, the subset of B that satisfies $\sigma > \psi(1) + c$ contains the triangle: $\{(\lambda, \sigma)\colon \gamma < \sigma < \lambda_c,\ \sigma < \lambda < \lambda_c\}$, which has positive measure. □

Appendix C. Pooling equilibria when types are exogenous

In this appendix, we show that there is a continuum of pooling equilibria in the economy described in Section 5.2 where each seller is endowed with the ability to produce only one quality. Recall that we assume $\alpha = 1$ in this economy with exogenous quality.

Let us first redefine an equilibrium. Because free entry of sites is no longer applicable, the expected profit that a seller can get in the market is not zero in general. Let Π_k be the expected profit for a type-k seller in the market. Moreover, the measure of sellers of each type in the market should add up to the exogenously given mass. Thus, a Bayesian Nash equilibrium can be defined similarly to Definition 3.1, with the following modifications: Part (i) of the definition is replaced with: $\Pi_k = \pi_\mu(p, k) = \max_{p' \in [0,1]} \pi_\mu(p', k)$ for every (p, k) such that $N(p, k) > 0$, and $\Pi_k = 0$ otherwise; and part (iv) contains an additional condition: for $k \in \{\lambda, 1\}$, if $\Pi_k = 0$, then $N(p, k) = 0$ for all $p \in [0, 1]$; if $\Pi_k > 0$, then $\sum_p N(p, k) = \omega_k$. As explained in Section 5.2, Restriction 1 and parts (b) and (c) of Lemma 3.2 still hold. Part (a) of Lemma 3.2 no longer holds, and neither does Restriction 2.

Assume $\lambda > \psi(1) + c$. Let p_p be a given number in $(\psi(1) + c, \lambda)$, to be calculated below. We establish that there is a non-empty parameter region in which the following pooling equilibrium exists: (a) only submarket p_p is active; (b) both types of sellers enter submarket p_p; and (c) buyers' beliefs are such that $\mu(p_p) = \omega_H$, $\mu(p) = 1$ for $p \geqslant \lambda$, and $\mu(p) = 0$ for all $p < \lambda$ with $p \neq p_p$. Notice that the belief $\mu(p_p) = \omega_H$ follows from the consistency requirement (iii) in Definition 3.1, and the belief $\mu(p) = 1$ for $p \geqslant \lambda$ follows from Restriction 1. Denote the average quality in submarket p_p as $\bar{k} = \omega_H + (1 - \omega_H)\lambda$.

We start by calculating p_p and the related variables. Define (p_p, θ_p) as follows:

$$(p_p, \theta_p) = \arg \max_{\psi(1)+c<p<\lambda} \frac{F(\theta)}{\theta} p, \quad \text{s.t. } F(\theta)(\bar{k} - p) = D, \tag{C.1}$$

where D is the market payoff to a buyer. The objective function in the above maximization problem is a seller's expected revenue in submarket p, and the constraint ensures that visiting the submarket is optimal for a buyer. If submarket p_p is indeed the only active submarket in the equilibrium, then the tightness in the submarket is $\theta_p = 1$. For the pooling equilibrium described above to be an equilibrium, it is necessary that p_p lies in the interior of $(\psi(1) + c, \lambda)$. Under this condition, the first-order conditions of the above maximization problem yield

$$p_p = \bar{k} F'(1)/F(1), \qquad D = \bar{k}\big[F(1) - F'(1)\big]. \tag{C.2}$$

A seller's expected revenue in submarket p_p is $\bar{k} F'(1)$, and the expected profit of entering the submarket for a type-k seller is $\Pi_k = \bar{k} F'(1) - \psi(k) - c$. The equilibrium requires $\Pi_k > 0$ for both $k = \lambda$ and $k = 1$. We combine this requirement on the expected profit and the requirement, $p_p \in (\psi(1) + c, \lambda)$, as

$$\lambda > \bar{k} F'(1)/F(1) \quad \text{and} \quad \bar{k} F'(1) > \psi(1) + c. \tag{C.3}$$

For the described pooling to be an equilibrium, we also need to show that there is no incentive for a seller to enter any submarket other than p_p. By the construction of p_p, it is clear that a seller (high- or low-quality) does not want to enter a submarket $p < \lambda$ with $p \neq p_p$. If a seller did enter such a submarket, the seller would be viewed by buyers as a low-quality seller according to the described beliefs, in which case the maximum expected revenue for the seller would be strictly lower than the one in submarket p_p. So, let us consider a deviation to a submarket $p \geqslant \lambda$. We have already explained in Section 5.2 that entering such a submarket is not optimal for a low-quality seller. Consider a high-quality seller. If a high-quality seller enters submarket $p \geqslant \lambda$, buyers will view the seller correctly as a high-quality seller (Restriction 1). The expected surplus for a buyer visiting such a submarket is $F(\theta)(1-p)$, where θ is the tightness in the submarket. There are two cases to analyze:

Case 1. $\lambda \geqslant \bar{\lambda}$, where $\bar{\lambda}$ is defined as

$$\bar{\lambda} \equiv 1 - \bar{k}\big[F(1) - F'(1)\big]. \tag{C.4}$$

Note that the right-hand side of (C.4) is equal to $1 - D$, where D is given in (C.2) as a buyer's expected surplus in the market. With $\lambda \geqslant \bar{\lambda}$, the expected surplus for a buyer visiting submarket $p \geqslant \lambda$ is at most equal to $F(\theta)(1-\lambda) \leqslant D$, where the equality holds if and only if $\theta = \infty$ and $p = \lambda$. In this case, it is not profitable for a high-quality seller to deviate to submarket $p \geqslant \lambda$.

Case 2. $\lambda < \bar{\lambda}$. In this case, there are feasible combinations of (p, θ) such that a buyer's expected surplus of visiting $p \geqslant \lambda$ is equal to D. Among all these combinations, the one that maximizes a high-quality seller's expected revenue solves

$$\max_{p \geqslant \lambda} \frac{F(\theta)}{\theta} p, \quad \text{s.t. } F(\theta)(1-p) = \bar{k}\big[F(1) - F'(1)\big]. \tag{C.5}$$

Denote the solution as $(\tilde{p}, \tilde{\theta})$. If $\tilde{p}$ is interior, i.e., if $\tilde{p} > \lambda$, we can prove that the deviation to $\tilde{p}$ is profitable for a high-quality seller.[19] Thus, when $\lambda < \bar{\lambda}$, a high-quality seller does not deviate to $p \geqslant \lambda$ only if $\tilde{p} = \lambda$. From the constraint in (C.5), it is clear that $\tilde{p} = \lambda$ if and only if $\tilde{\theta} = \theta_0$, where θ_0 is defined by

$$F(\theta_0) = \frac{\bar{k}}{1-\lambda}\big[F(1) - F'(1)\big]. \tag{C.6}$$

Substituting p from the constraint in (C.5), we can write the seller's expected revenue in submarket $p \geqslant \lambda$ as $\frac{1}{\theta}\{F(\theta) - \bar{k}[F(1) - F'(1)]\}$. This expected revenue attains the maximum at the corner $\tilde{\theta} = \theta_0$ if and only if

$$F(\theta_0) - \theta_0 F'(\theta_0) \geqslant \bar{k}\big[F(1) - F'(1)\big]. \tag{C.7}$$

Under this condition, the deviation to $\tilde{p}$ is not profitable if and only if

$$\frac{1}{\theta_0}\big\{F(\theta_0) - \bar{k}\big[F(1) - F'(1)\big]\big\} \leqslant \bar{k} F'(1). \tag{C.8}$$

[19] To see this, suppose $\tilde{p} > \lambda$. Then the first-order condition and the constraint in (C.5) imply: $\tilde{p} = \tilde{\theta} F'(\tilde{\theta})/F(\tilde{\theta})$ and $F(\tilde{\theta}) - \tilde{\theta} F'(\tilde{\theta}) = \bar{k}[F(1) - F'(1)]$. The second equation implies $\tilde{\theta} < 1$, because $[F(\theta) - \theta F'(\theta)]$ is strictly increasing and $\bar{k} < 1$. In this case, $[F(\tilde{\theta})/\tilde{\theta}]\tilde{p} = F'(\tilde{\theta}) > F'(1) > \bar{k} F'(1)$, where the two inequalities follow from $\bar{k} < 1$ and the fact that $F'(\theta)$ is strictly decreasing. That is, the seller's expected revenue is strictly higher in submarket $\tilde{p}$ than in submarket p_p.

Thus, when $\lambda < \bar{\lambda}$, a high-quality seller does not deviate to any submarket $p \geqslant \lambda$ if and only if (C.7) and (C.8) are satisfied.

To see how these conditions restrict λ, we express θ_0 as $\theta_0(\lambda)$. It is clear from (C.6) that $\theta_0'(\lambda) > 0$ for all $\lambda < \bar{\lambda}$, and that $\theta_0(\bar{\lambda}) = \infty$. Because $[F(\theta_0) - \theta_0 F'(\theta_0)]$ is a strictly increasing function of θ_0 and approaches 1 when $\theta_0 \to \infty$, then it is strictly increasing in λ for all $\lambda < \bar{\lambda}$ and approaches 1 when $\lambda \to \bar{\lambda}$. These features imply that there exists $\lambda_a \in (0, \bar{\lambda})$ such that (C.7) is equivalent to $\lambda \geqslant \lambda_a$. Similarly, because the left-hand side of (C.8) is strictly decreasing in θ_0 (see (C.7)), it is strictly decreasing in λ, and it approaches 0 when $\lambda \to \bar{\lambda}$. Thus, there exists $\lambda_b < \bar{\lambda}$ such that (C.8) is equivalent to $\lambda \geqslant \lambda_b$. Define $\lambda_0 = \max\{\lambda_a, \lambda_b\}$. Then, $\lambda_0 < \bar{\lambda}$. Moreover, when $\lambda < \bar{\lambda}$, (C.7) and (C.8) are both satisfied if and only if $\lambda \in [\lambda_0, \bar{\lambda})$.

Putting Case 1 and Case 2 together, we conclude that the deviation to $p \geqslant \lambda$ is not profitable for a high-quality seller if and only if $\lambda \geqslant \lambda_0$, where $0 < \lambda_0 < \bar{\lambda}$. The condition $\lambda \geqslant \lambda_0$ and (C.3) together define the parameter region in which the pooling equilibrium exists. This region is non-empty: we can pick $(\omega_H, \psi(1) + c)$ to satisfy the second condition in (C.3), and then set $\lambda \geqslant \max\{\lambda_0, \bar{k}F'(1)/F(1)\}$ to satisfy the remaining conditions.

Finally, we prove that there is a continuum of pooling equilibria. Consider pooling in submarket $\hat{p} = p_p - \varepsilon$, where p_p is constructed above and $\varepsilon > 0$ is small. Let buyers' beliefs be such that $\mu(\hat{p}) = \omega_H$, $\mu(p) = 0$ for all $p < \lambda$ with $p \neq \hat{p}$, and $\mu(p) = 1$ for $p \geqslant \lambda$. Similar to the procedure above, we can find a non-empty parameter region in which deviations to $p \geqslant \lambda$ are not profitable. In addition, a deviation to $p < \lambda$ with $p \neq \hat{p}$ induces buyers to view the seller as a low-quality seller. The maximized expected revenue from this deviation is $\max_{p<\lambda}[pF(\theta)/\theta]$ s.t. $F(\theta)(\lambda - p) = D$. Comparing this problem with (C.1), we can see that when ε is sufficiently small, the expected revenue from deviating to $p < \lambda$ with $p \neq \hat{p}$ is lower than staying in submarket $\hat{p}$. Thus, there is an interval of $\hat{p}$ near p_p such that pooling in submarket $\hat{p}$ is an equilibrium with the described beliefs.

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