SHOULD BUYERS OR SELLERS ORGANIZE TRADE IN A FRICTIONAL MARKET?*

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To answer the question in the title, this article characterizes the socially efficient organization of the market with search frictions. The efficient organization depends on the relative elasticity in the supply between the two sides of the market, the costs of participating in the market and organizing trade, and the (a)symmetry in matching. We also show that the social optimum can be implemented by a realistic market equilibrium where the organizers set up trading sites to direct the other side's search. The results provide a unified explanation for why trade has often been organized by sellers in the goods market, by buyers (firms) in the labor market, and by both sides in the asset market. The analysis also sheds light on how the efficient market organization can change with innovations such as e-commerce and just-in-time production. *JEL* Codes: D40, D60, D83.

I. Introduction

Search frictions impede trade. To mitigate these frictions, some individuals actively organize trade by setting up trading sites to direct other participants' search. The trading sites can be shops, job advertisements, websites, etc. In addition to the site cost, there are costs of participating in the market. To maximize social welfare in such a frictional market, should buyers or sellers organize trade? We address this question by characterizing the social optimum constrained by search frictions.

In reality, market organizers vary across markets. In the goods market, sellers set up shops and advertise to direct buyers' search. In the labor market, buyers of labor services (firms) organize trade by incurring the cost to post vacancies, while sellers (workers) search for jobs. In the asset market, buyer-organized trade and seller-organized trade coexist. How are these variations in the market organization related to search frictions and trading costs? Moreover, matching and trading technologies can change over time, as witnessed in the fast growth of e-commerce in the

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past two decades. How do these innovations affect the socially efficient organization of a market?

Given the prevalence of search frictions, questions like the ones above should have first-order importance in economics. However, they have been either made irrelevant by assumption or not addressed formally. In the Walrasian paradigm, the market organization is irrelevant because trade is assumed to be frictionless. The literature on firms and organizations, pioneered by Coase (1937), Williamson (1981), and Grossman and Hart (1986), assumes the market to be inefficient to determine the boundaries of firms, but it takes the market as given. In this article, we put the organization of the market at the center of the analysis. To address the question who should organize trade, we focus on the social planner's choice of the market organization under search frictions. This normative focus is also helpful for shedding light on what regulations are necessary for inducing the efficient organization if the market is inefficient. In the end, we show that the social optimum can be implemented by an equilibrium with price posting and directed search—a market mechanism commonly observed in reality.

It may not be obvious why the market organization can matter for social welfare, especially when search is directed. In the labor market, for example, Burdett, Shi, and Wright (2001) assume that firms post wages to direct workers' search, but Julien, Kennes, and King (2000) assume that workers post auctions with reserve wages to direct firms' search.[1] Given the market organization in these models, the equilibrium is socially efficient under the constraint of matching frictions. This result gives the impression that which side of the market organizes trade is immaterial for efficiency, as long as search is directed. However, this impression is false. Under both market organizations, match failures arise from the lack of coordination among searchers. Some searchers may apply to the same target, but only one of them is chosen to form a match. The difficulty of coordinating increases with the number of targets per searcher. Thus if the short side of the market organizes trade, the number of matches is higher, which increases social welfare (see Herreiner 1999). In these papers, the role of market organizers is captured by an asymmetry in the matching

1. Burdett, Shi, and Wright (2001) examine the goods market in the main sections of their paper, but they discuss the model's implications for the labor market in the concluding section.

function. The matching game generates the so-called urn-ball matching function. Switching the two sides of the market in this matching function yields different numbers of matches.

This example illustrates the importance of search frictions but misses several necessary ingredients for an analysis of the efficient market organization. First, which side of the market is short should be endogenous rather than fixed. It is necessary to trace the determinants of the efficient market organization to the fundamentals. Second, social welfare depends on the trading cost, not just the number of matches emphasized in the above example. Social efficiency may call for a compromise on the number of matches to economize on the trading cost, and a market organized by the long side may maximize social welfare sometimes. Third, the example uses a particular matching game that yields an asymmetric matching function. To understand the efficient market organization, it is necessary to analyze both symmetric and asymmetric matching functions. Moreover, a general matching function enables us to examine how the efficient market organization changes with the matching process.

The benchmark model in this article incorporates these ingredients, with homogeneous individuals on each side of the market. On one side, the supply of individuals is elastic and determined by competitive entry. The supply on the other side is relatively inelastic; for simplicity, we fix its measure at 1. Both sides face a cost to participate in the market. In addition, if an individual chooses to organize trade, he or she must also incur a site cost to set up a trading site. The site capacity is normalized to one per organizer. If an individual does not organize trade, he or she is a visitor. The measure of matches is given by a general matching function that allows for potentially asymmetric roles of organizers versus visitors, such as the ones in the foregoing example of directed search. In a match, one indivisible unit of a good or service is traded.

We analyze the planner's allocation that maximizes the sum of expected net utilities of all individuals under the constraints of search frictions and individual rationality. Because individuals on each side are homogeneous, the efficient allocation is to have only one side of the market organize trade. The planner chooses which side to organize trade and how many elastic individuals to enter the market. We characterize the social optimum under symmetric matching first in Section III and then under asymmetric matching in Section IV.

When the matching function is symmetric and the site cost is positive, the model generates three main predictions: (i) the short side of the market should organize trade; (ii) the elastic side is short if and only if the side's participation cost exceeds a threshold; (iii) a reduction in the site cost increases this threshold and, hence, increases the likelihood that the efficient organizers are on the inelastic side. Because matching is symmetric, these predictions have nothing to do with the earlier example that relies on asymmetric matching. Instead, they arise from the fact that an organizer incurs the site cost but a visitor does not. Because of this asymmetric effect, fewer elastic individuals enter the market if they are organizers than if they are visitors. The lower entry economizes on the total cost of trade at the expense of reducing the number of matches. When the elastic side's participation cost exceeds a threshold, economizing on the total cost of trade is the dominant consideration for efficiency. In this case, the elastic side should incur the site cost to organize trade, which puts them on the short side. When the elastic side's participation cost is below the threshold, increasing the number of matches is the dominant consideration for efficiency. In this case, the inelastic side should incur the site cost to induce more elastic individuals to enter the market, which again puts the organizers on the short side. A reduction in the site cost reduces the importance of the total cost of trade in the efficiency consideration and, hence, increases the threshold above which the elastic side should organize trade. If the site cost is zero, social welfare is independent of which side organizes trade, despite the existence of search frictions and participation costs.

The predictions accord well with observations in the goods market and the labor market. In the goods market, sellers are elastic; in the labor market, firms (buyers) are elastic. In both, an elastic individual incurs a substantial participation cost to establish a business or set up production. The site cost is also positive. Given these features, trade should be organized by sellers in the goods market and by firms in the labor market. Such market organizations have been prevalent in the form of shops maintained by sellers and of jobs advertised by firms. Also, because the efficient market organizers are on the short side, there is a welfare justification for why there are fewer job vacancies than there are unemployed workers. However, these predictions call for a distinction between the site cost and the participation cost, which is blurred in the literature (see Section III.B).

These predictions help us understand how the market organization can evolve with technological advances such as the growth of e-commerce. As online shops replace physical shops, the site cost and the participation cost fall significantly. Another advance is the increasing use of just-in-time production, which postpones part of the production cost from the participation cost to the post-match cost. In the labor market, sectoral changes move jobs from manufacturing to services where firms are less costly to set up and jobs are more flexible. By the third prediction, all these changes increase the likelihood that more goods will be made on demand instead of being made for order and that more job-wanted instead of help-wanted announcements will be advertised.

When the matching function is asymmetric, we define intuitively whether the function favors the short or the long side (see Section II.A) and link the asymmetry to the underlying meeting process (see Section IV). The additional prediction is as follows: if matches are generated by one-to-many meetings, such as the process underlying the urn-ball matching function, the market should be organized by the short side even when the site cost is zero; if matches are generated by one-to-one meetings and if the organizers have sufficiently lower search efficiency than the visitors, then the market should be organized by the long side. In Section IV, we use this prediction to shed light on the differences between the marriage market and the labor market. We also discuss how the market organization changes with technological innovations, such as online trading and trading platforms.

Section V introduces heterogeneity on the inelastic side. The new result is that markets organized by different sides can coexist when the elastic side's participation cost is intermediate. Applying this result to the asset market, the model yields the following prediction: asset sellers who have high liquidity needs organize one market to initiate trade, and asset sellers who have low liquidity needs participate as visitors in another market organized by buyers. We use this prediction to explain the trading pattern and the growth of the short-term loan market of repurchase agreements (repos).

In Section VI, we consider a realistic market mechanism where individuals compete to set up trading sites and post the terms of trade to direct the other side's search. We show that the market equilibrium implements the social optimum. In addition to internalizing matching externalities, competition with directed search induces the efficient organization of the market to emerge.

Matching frictions play a pivotal role in our analysis. If matching frictions did not exist, social welfare would be independent of which side organizes trade. A contrast is Taylor (1995), who compares prices posted by the two sides of a market without matching frictions. All equilibria in his paper yield the same social welfare.

This article is related to the literature on directed search pioneered by Peters (1991) and Montgomery (1991). However, this literature exogenously fixes one side of the market to direct search. For the goods market, the literature assumes that sellers direct buyers' search, for example, Peters (1991) and Burdett, Shi, and Wright (2001). For the labor market, the literature assumes that search is directed by firms in some papers (e.g., Moen 1997; Acemoglu and Shimer 1999; Burdett, Shi, and Wright 2001; Shi 2001; Galenianos and Kircher 2009), and by workers in other papers (e.g., Julien, Kennes, and King 2000). We endogenize the market organization to show that social efficiency calls for a specific side of the market to direct search.[2]

In a matching game, Herreiner (1999) demonstrates that the number of matches is higher if the short side of the market directs the other side's search. She fixes the number of participants on each side and the game generates the urn-ball matching function that favors the short side. As we explained earlier, it is necessary to endogenize the relative supply between the two sides of a market and use a general matching function. An illustration of this necessity is the case where matching is symmetric. When the relative supply between the two sides is endogenous, the efficient market organization is determinate, provided that the site cost is positive. When the relative supply is fixed as in Herreiner's model, welfare would be independent of which side organizes trade if the game were changed to yield a symmetric matching function. Moreover, with a general matching function, the efficient organizers are not always on the short side.[3]

2. The literature on directed search has often assumed that the search-directing side of the market is elastic in the supply. This is a possible cause of the false impression that social efficiency is independent of which side directs search. Under this assumption, a change in the market organization changes the model environment, which makes the two market organizations not comparable. We avoid this potential confusion by assuming that which side is elastic is independent of which side organizes trade.

3. Julien, Kennes, and King (2006) briefly examine how switching the roles of the two sides of a market in directed search can affect welfare, but they do not reach a clear conclusion.

The literature on undirected search has examined how an equilibrium changes with the search pattern. Burdett et al. (1995) compare the equilibrium where only one side of the market searches with the equilibrium where both sides search. Kultti et al. (2009) find that sufficiently unequal population between the two sides of a market is important for the equilibrium with one side searching to be robust to coalition deviations. These papers do not study the efficient market organization. In fact, if the supply of individuals is endogenous on at least one side, the equilibria in these models are generically inefficient because they fail to internalize matching externalities (see Hosios 1990).

In operations research, Alpern (1995) studies the least expected time for two individuals randomly placed in a region to find each other and shows that the symmetry in the region lengthens the expected time. Missing from this problem are the basic ingredients of an economic model, such as markets and the interactions among individuals. Finally, a literature on platform competition emphasizes network externalities (Rochet and Tirole 2003). We focus on search frictions instead. Throughout the analysis, we abstract from such externalities by assuming that the site cost is constant per site.

II. The Model

II.A. Model Environment

The economy lasts for one period and is populated by homogeneous and risk-neutral individuals on each side of the market.[4] One side of the market is elastic in the supply because the measure of individuals is determined by entry. The other side is relatively inelastic in the supply and the measure of individuals is fixed at 1. The elastic side is indexed by $i = e$ and the inelastic side by $i = n$. For brevity, we refer to an individual on side e as an elastic individual and an individual on side n as an inelastic individual. In the general description, we do not tie the elastic side to the supply or the demand side, because the tie can vary across markets. For specific examples, one can think of the elastic side as sellers in the goods market who compete to supply goods and as buyers (firms) in the labor market who compete to create vacancies. In

4. Heterogeneous individuals on the inelastic side are introduced in Section V and private information is discussed briefly in Section VII.

both examples, the inelastic side is not completely fixed. Instead, buyers in the goods market can delay search if prices are exceedingly high, and workers can choose between work and leisure. However, fixing the measure of individuals on the inelastic side is without loss of generality, because only the relative elasticity between the two sides matters for the results as shown in Online Appendix G.[5]

To participate in the market, an individual on side i must incur a cost $c_i \geqslant 0$. The utility of staying out of the market is zero. In a match, one unit of an indivisible good is transferred from one party to the other party for consumption. The utility of consumption net of the postmatch production cost is normalized to 1. In addition to the indivisible good, there is a divisible good that everyone can produce and consume. The marginal cost of producing and the marginal utility of consuming the divisible good are equal to 1. This good is used to transfer utilities between individuals.

Any individual can become a market organizer by incurring a cost $k \geqslant 0$ for a site. We refer to the collection of trades organized by side $i \in \{e, n\}$ as market i. The individuals trading with the organizers are called visitors. An organizer faces a capacity constraint on the number of sites, which is normalized to 1 for simplicity.[6] Thus, the measure of trading sites is equal to the measure of organizers. The ratio of elastic individuals to inelastic individuals, denoted θ, is determined by entry endogenously. Since the measure of inelastic individuals is normalized to 1, θ is also the measure of elastic individuals. Note that organizing trade is not necessarily the same as posting the price. Although the two actions are often related in reality, it is conceivable that market organizers can create sites but allow visitors to name the price.

The matching process in a market is frictional. In general, an organizer and a visitor may contribute differently to the match formation, and the relative role switches with the market

5. There, we introduce search effort on the inelastic side to make the supply partially elastic and prove that the results do not change. We thank an editor for suggesting this extension.

6. Online Appendix F analyzes the effect of the site capacity. Also, the capacity constraint can be endogenized, although we do not carry out the exercise. If the marginal cost of a site is sufficiently increasing, the main results of this article continue to hold. On the other hand, if the marginal cost of a site is constant or decreasing, then the efficient allocation is the uninteresting outcome that only one organizer participates in the market to create all the sites needed for trade.

organization. For example, the organizers may direct visitors' search. To capture the role of the organizers consistently between different market organizations, we put the measure of the organizers always as the first argument in the matching function and the measure of visitors as the second argument. Thus, the measure of matches is $M(\theta, 1)$ in market e and $M(1, \theta)$ in market n, where M is a matching function with constant returns to scale. Denote $F(\theta) \equiv M(\theta, 1)$. Then, in market e, the site-visitor ratio is θ, the matching probability is $F(\theta)$ for a visitor (an inelastic individual) and $\frac{F(\theta)}{\theta}$ for a site (an elastic individual). In market n, the site-visitor ratio is $\frac{1}{\theta}$, the matching probability is $F(\frac{1}{\theta})$ for a visitor (an elastic individual) and $\theta F(\frac{1}{\theta})$ for a site (an inelastic individual).

Elastic individuals are on the short side of the market if and only if $\theta < 1$. We say that a matching function M is symmetric if $M(\theta, 1) = M(1, \theta)$ for all $\theta \geqslant 0$, favors the short side if $M(\theta, 1) > M(1, \theta)$ is equivalent to $\theta \in (0, 1)$, and favors the long side if $M(\theta, 1) > M(1, \theta)$ is equivalent to $\theta \in (1, \infty)$.[7] By the definition of F, it is clear that $M(\theta, 1) > M(1, \theta)$ if and only if $F(\theta) > \theta F(\frac{1}{\theta})$, and so (a)symmetry of the matching function can be defined equivalently with F. Note that the site cost and the possible asymmetry in the matching function are the defining features of an organizer in this model.

It is crucial to distinguish the site cost from the participation cost. All individuals need to incur the participation cost, but only the organizers incur the site cost. Thus, any trading cost that can be avoided by changing from an organizer to a visitor is a site cost. Conversely, any trading cost that must be incurred independently of whether an individual organizes trade is a participation cost. Clear examples of the site cost are the costs to maintain a shop, advertise, and maintain a vacancy, because a visitor does not incur such costs. In contrast, creating a job opening in the first place is a participation cost to a firm because the firm cannot avoid it by being a visitor instead of an organizer. Less clear is a seller's cost to set up a shop. Although it may be natural to regard this cost as a seller's participation cost, it is a site cost if the seller can avoid it by visiting shops created by buyers instead. Note that the participation cost also includes part of the search cost. For example, the literature specifies an unemployed worker's search

7. The three cases are not exhaustive. It is possible that a matching function is not symmetric but favors neither side.

cost to include the cost of staying in the labor force, which is a participation cost.[8]

We impose the following assumption on the function $F(\theta) = M(\theta, 1)$:

ASSUMPTION 1. $F(\theta) \in [0, 1]$ and $\frac{F(\theta)}{\theta} \in [0, 1]$ for all $\theta \in [0, \infty)$. For all $\theta \in (0, \infty)$, F is strictly concave and twice continuously differentiable, with $F > 0$, $F' > 0$, and $F - \theta F' > 0$. Moreover, $A \equiv F'(0) \in (0, 1]$, $F(\infty) = 1$, $F(0) = \lim_{\theta\to\infty} \theta F'(\theta) = 0$.

This assumption is standard. In particular, $F' > 0$ and $F - \theta F' > 0$ require that, as the number of sites per visitor increases, the matching probability should increase for a visitor and decrease for a site.[9] Also, all proper matching functions should have the property that the matching probabilities do not exceed 1. This property implies $A \equiv F'(0) \leqslant 1$, which is listed in Assumption 1 to facilitate the reference. To see why, recall that the matching probability for an elastic individual is $\frac{F(\theta)}{\theta}$ in market e. Because $\frac{F(\theta)}{\theta} \leqslant 1$ for all θ, then $F'(0) = \lim_{\theta\to 0} \frac{F(\theta)}{\theta} \leqslant 1$. Any matching function that violates $F'(0) \leqslant 1$ is improper (e.g., the Cobb-Douglas function), and it can be made proper by redefining $F'(\theta) = \min\{F(\theta), \theta, 1\}$. However, the redefined function fails to be differentiable at $\theta_0 < 1$ such that $F(\theta_0) = \theta_0$. Although the analysis can be modified to deal with such nondifferentiability, the modification is cumbersome and omitted. Moreover, note that a necessary but not sufficient condition for a matching function to be symmetric is $A = 1$ (see Lemma 2 in Appendix A).

The three matching functions in the following example satisfy Assumption 1 and have been widely used in the literature:

EXAMPLE 1. The urn-ball matching function yields $F(\theta) = \theta[1 - e^{-\frac{1}{\theta}}]$. This function has $A = F'(0) = 1$, favors the short side,

8. Other parts of the search cost are in k instead of c. For example, some time ago sellers of vacuum cleaners carried samples to sell door to door. If sellers had stayed put to wait for buyers to visit, as they do now, they would have saved the carrying cost, but then buyers would have had to incur the search cost. Although the heterogeneity in the site cost between the two sides is interesting, we abstract from it for simplicity. It is predictable that a lower site cost gives a side an advantage in organizing trade. Burdett et al. (1995) explore the importance of search costs for the use of money.

9. Note that $F(\infty) = 1$ implies $\lim_{\theta\to\infty} [\frac{F(\theta)}{\theta}] = 0$ and that $\lim_{\theta\to\infty} \theta F'(\theta) = 0$ implies $F'(\infty) = 0$.

and can be derived as the outcome of a large directed-search game (see Burdett, Shi, and Wright 2001). The Dagum (1975) function is $F(\theta) = [(A\theta)^{-\rho} + 1]^{-\frac{1}{\rho}}$ with $\rho \in (0, \infty)$ and $A \in (0, 1]$, where A is an organizer's search efficiency relative to a visitor's. This function is symmetric if $A = 1$ and favors the long side if $A < 1$. The special case $\rho = 1$ is the telephone matching function, $F(\theta) = \frac{A\theta}{A\theta+1}$, which can be derived as the outcome of a bilateral matching game, for example, Burdett et al. (1995).

II.B. Planner's Problem

The social planner maximizes social welfare defined as the sum of all individuals' expected net utilities, subject to individual rationality (participation) constraints.[10] The planner can make transfers between the two sides. However, the planner faces the same search frictions the market does. Precisely, the planner takes the matching function as given and must treat all identical sites or visitors symmetrically. Although the social planner allocates the individuals to the matching process directly without resorting to prices, we continue to use the term "market" for convenience. The subscript i indexes the variables in market i. We formulate the planner's problem for each market i under the assumption that the planner can create only market i. Then we argue that the welfare comparison between the two markets is valid even if the planner can create markets e and n simultaneously.

Consider the case where the planner creates only market e. The measure of sites is equal to the measure of elastic individuals, which is θ_e. The sum of elastic individuals' costs of participation and sites is $(c_e + k)\theta_e$. Inelastic individuals' participation costs sum up to c_n. Because the matching probability of an inelastic individual is $F(\theta_e)$, total expected utility generated by all trades is $F(\theta_e)$. Thus, social welfare is equal to $w_e(\theta_e)$ where

$$w_e(\theta) \equiv F(\theta) - (c_e + k)\theta - c_n. \tag{1}$$

The planner chooses θ_e to maximize $w_e(\theta_e)$, subject to individual rationality constraints on each side that the expected surplus of participating is nonnegative. If social welfare is positive, the planner can use transfers to ensure that individual rationality

10. Most of the results continue to hold when the two sides of the market have different welfare weights. See Proposition 2.

constraints do not bind. Thus, the optimal choice of θ in market e is $\theta_e(c_e + k)$ where[11]

$$\theta_e(x) \equiv F'^{-1}(x) \text{ for all } x \geqslant 0. \tag{2}$$

Maximized social welfare in market e is

$$W_e = f(c_e + k) - c_n, \tag{3}$$

where f and h are defined as

$$\begin{aligned} h(\theta) &\equiv F(\theta) - \theta F'(\theta) \quad \text{for } \theta \in [0, \infty), \\ f(x) &\equiv h\left(F'^{-1}(x)\right) \quad \text{for all } x \geqslant 0. \end{aligned} \tag{4}$$

Similarly, if the planner creates only market n, then the measure of sites is equal to the measure of inelastic individuals, which is 1. The sum of inelastic individuals' costs of participation and sites is $(c_n + k)$. The measure of elastic individuals is θ_n, and their costs sum up to $c_e\theta_n$. Because the matching probability of an inelastic individual is $\theta_n F(\frac{1}{\theta_n})$, social welfare is equal to $w_n(\theta_n)$ where

$$w_n(\theta) \equiv \theta F\left(\frac{1}{\theta}\right) - c_e\theta - (c_n + k). \tag{5}$$

The function $w_n(\theta)$ is maximized at $\theta = \theta_n(c_e)$ where

$$\theta_n(x) \equiv \frac{1}{h^{-1}(x)} \text{ for all } x \geqslant 0. \tag{6}$$

Because $F'(h^{-1}(x)) = f^{-1}(x)$ for all $x \geqslant 0$, which is proven in Lemma 1, maximized social welfare in market n is

$$W_n = f^{-1}(c_e) - (c_n + k). \tag{7}$$

Market i is viable if $W_i > 0$. Market e dominates market n in social welfare if $W_e > W_n$, and market n dominates market e if

11. To simplify the expressions, we extend the inverse of any monotone function $L(z)$ outside the range of L as follows. Let the domain of L be $[z_1, z_2]$ and the range be $[x_1, x_2]$. If L is an increasing function, define $L^{-1}(x) = z_1$ for all $x < x_1$ and $L^{-1}(x) = z_2$ for all $x > x_2$. If L is a decreasing function, we define $L^{-1}(x) = z_2$ for all $x < x_1$ and $L^{-1}(x) = z_1$ for all $x > x_2$. This definition of the inverse extends $L^{-1}(x)$ from the domain $[x_1, x_2]$ to all x. In particular, $F'^{-1}(x) = \infty$ for all $x < 0$ and $F'^{-1}(x) = 0$ for all $x > A$.

$W_n > W_e$. If $W_e = W_n$, the two markets are welfare equivalent. If only one market can be created, the planner will create the dominant market. For the efficient allocation to have active trading, at least one market should be viable. Also, net utility of consumption should be high enough to cover all trading costs. We list these assumptions below.

ASSUMPTION 2. $\max\{W_e, W_n\} > 0$ and $0 \leqslant k < 1 - c_e - c_n$.

The foregoing characterization of the efficient allocation is valid even if the planner can create the two markets simultaneously. The efficient market is still the one with higher welfare. To see this, suppose that inelastic individuals are divided between markets e and n. Reinterpret W_i as social welfare per inelastic individual in market i. If $W_e > W_n$, the planner can move inelastic individuals from market n to market e and increase the measure of elastic individuals in market e to keep the ratio θ_e unchanged. Because the matching technology has constant returns to scale, this move does not change the matching probabilities in market e. For each inelastic individual moved to market e, welfare increases by $(W_e - W_n)$. The planner can continue to increase social welfare this way until all inelastic individuals are moved to market e. Similarly, if $W_e < W_n$, the planner can increase social welfare by moving all inelastic individuals from market e to market n.

Denote $G(c_e, k) = W_n - W_e$ where

$$G(c,k) \equiv f^{-1}(c) - k - f(c+k). \tag{8}$$

Then, market n dominates market e if and only if $G(c_e, k) > 0$. Define c_d as the unique solution to:

$$f(c_d + k) = c_d. \tag{9}$$

Note that $G(c_d, k) = 0$, and so the two markets are welfare equivalent if $c_e = c_d$.

ASSUMPTION 3. Regularity: $G_c'(c,k)$ has the same sign as $G_c'(c_d,k)$ at all interior solutions of c to $G(c, k) = 0$; if matching is asymmetric, then $G_c'(c_d,k) \neq 0$ for all $k \geqslant 0$.

As shown in Lemma 3 in Appendix A, the regularity condition ensures c_d to be the unique interior solution of c to $G(c,k) = 0$ under all symmetric matching functions for $k > 0$ and under all asymmetric matching functions for $k \geqslant 0$. Lemma 3 also proves that the regularity assumption is satisfied under all symmetric matching functions and under the urn-ball matching function. When the regularity assumption is violated, there can be multiple interior solutions of c to $G(c, k) = 0$, which are examined in Appendix C.

Matching frictions are necessary for the market organization to be relevant for social welfare in this model. To demonstrate, consider the frictionless matching function $M(\theta, 1) = \min\{\theta, 1\}$, under which the short side of the market is matched with probability 1. In both market e and market n, welfare is maximized at $\theta = 1$, and maximized welfare is the same in the two markets. Moreover, the efficient allocation in the frictionless economy can be approached as the limit of the efficient allocation in a sequence of economies with matching frictions. To see this, consider a sequence of economies where the matching function is the Dagum function in Example 1 with $\rho_j \in (0, \infty)$ and $A = 1$. In the limit $\rho_j \to \infty$, the matching function approaches the frictionless matching function. For each ρ_j, let the efficient allocation be θ_{ij} in market $i \in \{e, n\}$. With equations (2) and (6), it can be verified that $\theta_{ej} \to 1$ and $\theta_{nj} \to 1$ as $\rho_j \to \infty$. We summarize these findings:

REMARK 1. When matching is frictionless, market e and market n are welfare equivalent, despite the existence of participation costs and site costs. Moreover, the efficient allocation under frictionless matching can be approached as the limit of the efficient allocation under matching frictions as such frictions vanish.[12]

III. EFFICIENT MARKET ORGANIZATION UNDER SYMMETRIC MATCHING

III.A. Main Results and Intuition

The following theorem is proven in Appendix B:

THEOREM 1. Assume that the matching function is symmetric and define $c_d(k)$ by equation (9). If $k > 0$ and $c_e \neq c_d$, the short side of the market should organize trade. The short side is elastic if $c_e > c_d$ and inelastic if $c_e < c_d$, where $c_d(k)$ is a decreasing

12. Although the allocation is continuous at the frictionless limit in our model, such continuity cannot be presumed in general. Rubinstein and Wolinsky (1985) and Wolinsky (1990) are well-known examples in which frictional allocations fail to approach the frictionless allocation as the frictions vanish.

function. If $k = 0$ or $c_e = c_d$, social welfare is independent of which side organizes trade.

Figure I illustrates this theorem in the parameter space (c_e, c_n) for any given $k > 0$, where $A = 1$. In the positive quadrant, market e is viable below the curve $f(c_e + k)$, and market n is viable below the curve $f^{-1}(c_e) - k$. The two curves cross each other at $c_e = c_d$. For $c_e > c_d$, the curve $f(c_e + k)$ lies above $f^{-1}(c_e) - k$, and so market e dominates market n. For $c_e < c_d$, the curve $f(c_e + k)$ lies below $f^{-1}(c_e) - k$, and so market n dominates market e. For all $c_e \in [0, A - k]$ with $c_e \neq c_d$, the ratio of sites to visitors is less than 1 under the efficient market organization. When $k = 0$, the curves in Figure I coincide. Finally, when k increases, the curves shift down toward the origin but still intersect with each other on the 45-degree line, resulting in a lower threshold c_d.

A positive site cost tilts the favor toward the short side as market organizers despite symmetric matching. However, which side of the market is relatively short is endogenous. Elastic individuals are on the short side if $c_e > c_d$, and on the long side if $c_e < c_d$.

What is the explanation for these results? Because matching is symmetric, the intuition cannot be the one given in the introduction for the matching game studied by Herreiner (1999), Burdett, Shi, and Wright (2001), and Julien, Kennes, and King (2000). Instead, the key to the explanation is that the site cost affects the entry of elastic individuals differently under the two

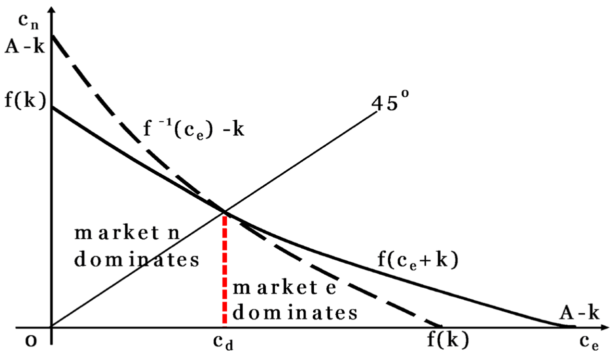

FIGURE I

Efficient Market Organization When the Matching Function Is Symmetric and $k > 0$

organizations. As a visitor, an elastic individual incurs only the participation cost c_e. As an organizer, an elastic individual incurs the site cost k in addition to the participation cost. Thus, a smaller measure of elastic individuals enter the market when they are organizers than when they are visitors. The smaller amount of entry reduces the total cost of trade but also reduces the measure of matches. For social welfare, there is a trade-off between these two dimensions. If the participation cost on the elastic side is high in the sense $c_e > c_d$, the cost saving dominates the consideration of the measure of matches. To save the cost, elastic individuals should incur the site cost to organize trade so that they do not enter the market excessively, which puts them on the short side. If the participation cost on the elastic side is low in the sense $c_e < c_d$, increasing the measure of matches dominates the cost-saving consideration. In this case, inelastic individuals should incur the site cost to induce more elastic individuals to enter the market, which again puts the organizers on the short side.[13]

With the above explanation, it is easy to understand why the threshold c_d decreases in the site cost. A higher site cost increases the importance of economizing on the total cost. In this case, it is more likely that elastic individuals should organize trade. That is, c_d is lower so that it is more likely for $c_e > c_d$ to occur. However, when $k = 0$, the market organization is irrelevant for social welfare. When $k = 0$, the marginal cost of entry on the elastic side is equal to the participation cost; hence, it is independent of which side organizes trade. The social marginal benefit of entry is to increase the inelastic side's matching probability. Because matching is symmetric, this marginal benefit is also independent of which side organizes trade. Thus, when $k = 0$, the two market organizations induce the same amount of entry of elastic individuals and yield the same welfare.

Reflecting the asymmetric effect of the site cost on the two markets, there are parameter regions in which one market is viable but the other is not. In Figure I, market e is viable but market n is not if the parameters lie above the curve $f^{-1}(c_e) - k$ and below

13. The trade-off between the trading volume and the total cost implies that social welfare is related ambiguously to the trading volume. If $c_e < c_d$, the efficient market (i.e., market n) increases the trading volume relative to market e. However, if $c_e > c_d$, the efficient market (i.e., market e) can reduce the trading volume relative to market n. Similarly, the relationship between social welfare and the market size depends on whether $c_e < c_d$, where the market size is the measure of individuals in the market.

the curve $f(c_e + k)$ for $c_e > c_d$. Market n is viable but market e is not if the parameters lie above the curve $f(c_e + k)$ and below $f^{-1}(c_e) - k$ for $c_e < c_d$.

In the next subsection, we map the results into observations in the goods market and the labor market. We discuss the asset market in Section V. For the mapping, it is useful to list the main results in Theorem 1 as follows:

- **Prediction 1**: Elastic individuals should organize trade if their participation cost is high.
- **Prediction 2**: The organizers are on the short side of the market.
- **Prediction 3**: A sufficiently large reduction in the site cost or the elastic side's participation cost can change the efficient organizers from the elastic side to the inelastic side.

III.B. Implications for the Goods Market and the Labor Market

For the mapping between the model's predictions and observations, it is important to recall the general interpretation that the inelastic side of a market is not fixed but inelastic only relative to the other side, as discussed in Section II.A.

1. **The Goods Market**. Sellers are usually on the elastic side to compete for buyers, and they incur substantial participation costs. To participate in the market, a seller acquires knowledge of the business and maintains the relationship with distributors and wholesalers. In addition, a seller incurs the cost to obtain some products for inventory and display. For a buyer, the search cost is the main cost of participation. In this market, a trading site can be a shop, a website for the product, or a membership in a trading platform. The site cost includes the cost to maintain a site and to advertise the product. Part of the cost of setting up a shop is also a site cost if an individual can avoid the cost by being a visitor. With this description, Predictions 1 and 2 state that sellers should organize trade and be on the short side. Both predictions accord well with observations in the goods market. Shops maintained by sellers have been the main trading form in the retail sector. Buying shops are much less common, perhaps because such shops are less efficient for trade. The dominance of seller-organized trade extends from the product market to services. Most services have been advertised by providers instead of customers.

2. **The Labor Market**. Firms (buyers) are usually on the elastic side to create vacancies and compete for workers (sellers). A firm faces large participation costs, such as the cost to set up its operation. The cost of creating a job opening is also a participation cost, instead of a site cost, because the firm needs to create a job regardless of whether the firm organizes trade. The worker's participation cost is the search cost. A trading site consists of a job advertisement and the resources devoted to recruiting. Part of the site cost is the cost to maintain a vacancy, which, in principle, differs from the cost to create a job. With this description, Prediction 1 is consistent with the fact that firms maintain vacancies and advertise jobs.[14] Prediction 2 is consistent with the evidence that the ratio of vacancies to unemployed workers is less than 1. This ratio has been about 0.7 in the U.S. data on Job Openings and Labor Turnover Survey and the data on the Help-Wanted Index (see Pissarides 2009).

Note that the search literature has often assumed that sellers organize trade in the goods market and firms organize trade in the labor market (e.g., Diamond 1982; Mortensen 1982; Pissarides 2000). At first glance, our analysis seems to justify this assumption on efficiency grounds. A closer look reveals the opposite. A typical model sets the participation cost to zero and assumes a flow cost to maintain a post or a vacancy, which is a site cost. In such a model, $c_e = 0 < c_d(k)$, so trade should be organized by inelastic individuals, that is, by buyers in the goods market and by workers in the labor market. This is opposite to the market organization commonly observed in reality. To make the model consistent with efficiency, the literature should introduce a sufficiently high participation cost on the elastic side to generate $c_e > c_d$ and distinguish this cost from the site cost.

3. **The Evolution of the Efficient Market Organization**. By changing c_e or k, innovations can change the efficient market organization (Prediction 3). This evolution of the efficient

14. Lawyers, freelancers, and contractors actively advertise their services. However, they should be interpreted as sellers of services instead of workers. The labor market may have a tighter capacity constraint than the goods market, because a vacancy is typically filled by only one worker. Although this capacity constraint may affect the market organization, the effect is not clear. Even in businesses with severe capacity constraints, such as restaurants, sellers are often the organizers of trade. Moreover, the trading pattern can change over time without obvious changes in the capacity constraint, as we alluded to in footnote 8 with the example of vacuum cleaners.

organization can be traced out in Figure I. Suppose that c_e is so high initially that the economy lies in the parameter region above the curve $f^{-1}(c_e) - k$ and below the curve $f(c_e + k)$. In this region, only market e is viable. If k is fixed while c_e falls to the left of the curve $f^{-1}(c_e) - k$, market n becomes viable but is still dominated by market e. If c_e falls further to the left of $c_d(k)$, the efficient organization changes to market n. Similarly, if the site cost k is so high initially that $c_d(k) < c_e$, market e is efficient. Because k decreases while c_e is fixed, the curves in Figure I shift up. Their intersection moves up along the 45-degree line, resulting in a higher threshold $c_d(k)$. If the fall in k is sufficiently large so that $c_d(k) > c_e$, then market n becomes efficient.

In the goods market, new technologies reduce the site cost by enabling sellers to keep inventory at a lower cost and lower depreciation than before. They can also reduce sellers' participation cost by reducing the amount of goods that need to be purchased in advance of sales. A related but different innovation is the adoption of just-in-time production that shifts the production cost from the prematch stage to the postmatch stage, which is studied in detail in Section III.C. In addition, regulatory changes can reduce the cost of setting up a business, and new information technologies can reduce a seller's cost of learning about the trade. All these changes have the tendency to move the market from seller-organized trade to buyer-organized trade. In the labor market, sectoral changes move firms from manufacturing to services that require smaller costs to set up and maintain. For example, a job in software design is easier to set up and more flexible than a job on an assembly line. Reflecting this contrast, an assembly-line worker rarely advertises his or her labor service, but a software designer might do so.

Online trade is a prominent example of the reduction in the site cost and the participation cost. Setting up and maintaining a physical shop can be very costly. In comparison, online stores are much less costly to create and monitor. As e-commerce develops, it may become increasingly common for buyers to specify their demand on websites and sellers to search to meet such demand. This implication is also relevant for the teaching service of subjects for which the site cost and the instructors' participation cost are low. For such subjects, learners may post their needs online while instructors search. Similarly, posting jobs online can significantly reduce the vacancy cost, so we may see an increase in the advertisements for job-wanted relative to help-wanted.

III.C. Just-in-Time Production

The technology of just-in-time production enables a seller to shift part of the participation cost from the prematch stage to the postmatch stage. How does this technology affect the efficient market organization? This question requires a separate analysis because the shift in the cost is not just a reduction in c_e—it also changes the ex post match surplus. Precisely, let $\delta \in [0, 1)$ be the fraction of an elastic individual's participation cost postponed to the postmatch stage so that the individual's participation cost becomes $(1 - \delta)c_e$. Utility of consumption net of the postmatch production cost is $U = 1 - \delta c_e$.[15] Let $C_i U$ be the participation cost of an individual on side i and KU the site cost, so that C_i and K are the costs normalized by U. Then,

$$C_e = \frac{(1-\delta)\,c_e}{1-\delta c_e},\; C_n = \frac{c_n}{1-\delta c_e},\; K = \frac{k}{1-\delta c_e}. \tag{10}$$

After replacing (c_e, c_n, k) by (C_e, C_n, K), the analysis in Section II.B is valid for all $\delta \in [0, 1)$. The efficient ratio of elastic to inelastic individuals is $\theta_e(C_e + K)$ in market e and $\theta_n(C_e)$ in market n. Welfare per inelastic individual is W_i in market i, where

$$W_e = \left[f\,(C_e + K) - C_n\right] U$$
$$W_n = \left[f^{-1}\,(C_e) - C_n - K\right] U.$$

Market n dominates market e if and only if $W_n > W_e$, that is, if and only if $G(C_e, K) > 0$ where G is defined in equation (8).

To simplify the analysis, assume $k > 0$. Adapting the proof of Theorem 1, we have $G(C_e, K) > 0$ if and only if $C_e \in (0, C_d(K))$, where C_d solves equation (9). Moreover, $C_e < C_d(K)$ can be written as $c_e < c_d$, where c_d now denotes the unique solution to

$$\frac{(1-\delta)\,c_d}{1-\delta c_d} = C_d\left(\frac{k}{1-\delta c_d}\right).$$

Denote $f_e(c_e, \delta) = (1 - \delta c_e)f(C_e + K)$ and $f_n(c_e, \delta) = (1 - \delta c_e)[f^{-1}(C_e) - K]$, where C_e and K are functions of (c_e, k) defined by (10). Since $W_e = f_e(c_e, \delta) - c_n$ and $W_n = f_n(c_e, \delta) - c_n$, then market n dominates market e if and only if $f_n > f_e$.

15. The joint surplus U is independent of which side pays the postponed cost, δc_e. So are social welfare and the efficient market organization.

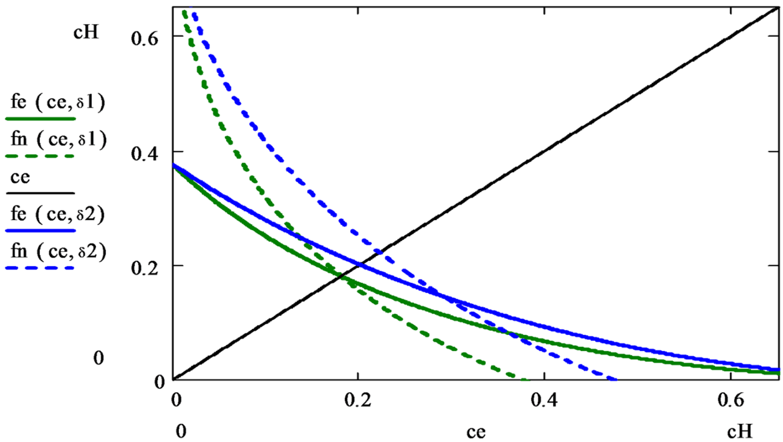

FIGURE II

The Market Organization When δ Fraction of c_e Is Shifted to Postmatch

Figure II depicts the effect of increasing δ from $\delta 1 = 0$ to $\delta 2 = 0.5$, with $k = 0.015$. It uses the telephone matching function in Example 1 with $A = 1$. After the increase in δ, both curves f_e and f_n shift up, so each market organization becomes viable in a wider region of the parameters (c_e, c_n). Social welfare increases under each market organization. Moreover, the curve f_n shifts up by more than the curve f_e. The new intersection between the curves is below the 45-degree line, and the threshold c_d increases.[16] Therefore, the delay of the production cost to postmatch increases the likelihood that the market organized by the inelastic side is efficient. This effect contrasts with a reduction in c_e alone, which does not change c_d, and with a reduction in k, which moves up the intersection between the curves along the 45-degree line.

The increase in δ improves social welfare by saving the cost δc_e when an elastic individual fails to match. This cost saving increases the gain to an elastic individual, induces higher entry of elastic individuals, and increases the measure of matches. To learn why an increase in δ benefits market n more than market e, recall that the trading cost to an elastic individual in market e

16. For any $\delta > 0$, the intersection between $f_e(c_e, \delta)$ and $f_n(c_e, \delta)$ continues to lie on the 45-degree line in the (C_e, C_n) diagram. However, since $C_n = C_e$ implies $c_n = (1 - \delta)c_e$, the intersection between the curves in the (c_e, c_n) diagram lies below the 45-degree line.

consists of both the participation cost and the site cost. Because the site cost does not change with δ, the delay in an elastic individual's participation cost reduces the individual's expected cost of trade less than one for one in market e. In contrast, in market n, the trading cost to an elastic individual consists of only the participation cost, so the delay in the individual's participation cost reduces the expected cost of trade one for one. Thus, the delay increases welfare by more in market n than in market e. As δ increases sufficiently, more products will be made on spot in a market organized by the inelastic side.

IV. Asymmetric Matching and Trading Platforms

Given the rich results obtained so far under symmetric matching, why is it useful to analyze asymmetric matching? There are several reasons. First, matching is asymmetric in well-known environments, such as the urn-ball matching function in Example 1. Second, as explained later, asymmetry in the matching function reflects fundamental features of the matching process, and examining the asymmetry can enhance the understanding of why the market organization can differ across markets. Third, examining asymmetric matching can help us understand how the efficient organization changes over time in a given market as the relative search efficiency between the two sides changes. For example, parameter A in the telephone matching function in Example 1 is the relative search efficiency of market organizers to visitors. A decrease in A from 1 changes the matching function from a symmetric function to one that favors the long side. This change may lead to a different market organization. The analysis of asymmetric matching also leads to a discussion of trading platforms.

The following theorem is proven in Appendix B:

Theorem 2. Assume that the matching function is asymmetric. (i) If the matching function favors the short side, efficient organizers are on the short side for all $k \geqslant 0$ and $c_e \neq c_d$. (ii) If the matching function favors the long side, there exist k_a and k_b defined in Appendix B, with $k_b > k_a \geqslant 0$, such that efficient organizers are on the short side if $k > k_b$ and on the long side if $k < k_a$.[17] In both (i) and (ii), elastic individuals are on the

17. If $k_a \leqslant k \leqslant k_b$ and the matching function favors the long side, the regularity condition in Assumption 3 can be violated. Appendix C analyzes the efficient organization in this case.

short side if $c_e > c_d$ and on the long side if $c_e < c_d$, where the threshold $c_d(k)$ decreases in k.

Asymmetric matching affects the two cutoff levels k_a and k_b. When the matching function favors the short side or does not favor the long side strongly (i.e., when $k_b < k$), Figure I is still valid for $k \geqslant 0$. However, when the matching function favors the long side strongly so that $k_a > k$, the efficient organization is depicted in Figure III, where the relative position of the two curves $f(c_e + k)$ and $f^{-1}(c_e) - k$ is switched from Figure I. For $c_e > c_d$, the curve $f(c_e + k)$ lies below the curve $f^{-1}(c_e) - k$, in which case market n dominates market e. For $c_e < c_d$, the curve $f(c_e + k)$ lies above the curve $f^{-1}(c_e) - k$, in which case market e dominates market n. For all $c_e \neq c_d$, there are more sites than visitors under the efficient organization.

Under asymmetric matching, the short side of the market is determined by the same consideration as under symmetric matching. That is, the social marginal benefit of entry by an elastic individual should be equal to the social marginal cost of entry. The asymmetry in the matching function affects the threshold, c_d, by affecting the marginal benefit of entry. Given c_d, it is still true that the elastic side is on the short side if and only if the side's participation cost exceeds c_d. In contrast to symmetric matching, the short side is not always the efficient side to organize the market.

If the matching function favors the short side, the asymmetry in the matching function reinforces the site cost to favor the short

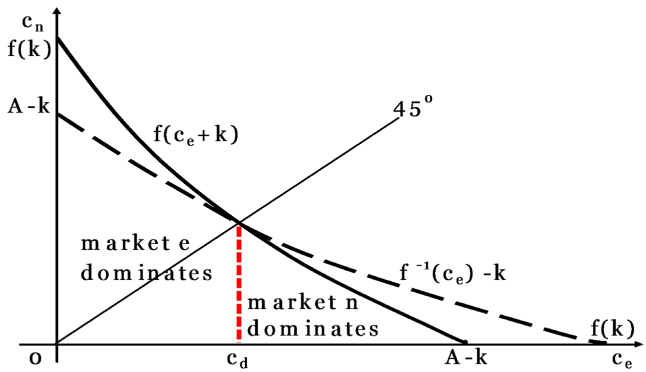

FIGURE III

Efficient Market Organization When the Matching Function Strongly Favors the Long Side

side as the efficient organizers of the market. In this case, the short side should organize trade even when the site cost is zero. This result contrasts with symmetric matching under which the two organizations are welfare equivalent when $k = 0$.

If the matching function favors the long side, the asymmetry in the matching function and the site cost have opposite effects on the efficient organization. If the matching function does not favor the long side strongly, that is, if $k_b < k$, then the site cost dominates the asymmetry in the matching function. In this case, the short side should organize trade, and other results are qualitatively similar to those under symmetric matching. If the matching function favors the long side strongly, that is, if $k_a > k$, then the asymmetry in the matching function dominates the site cost. In this case, the long side should organize trade. The long side is inelastic if $c_e > c_d$ and elastic if $c_e < c_d$, as in Figure III.

The asymmetry in the matching function reflects fundamental features of the matching process. To relate the market organization to these features, it is useful to contrast the microfoundations of the urn-ball function and the telephone matching function. For this purpose, we treat a match as a meeting process followed by selection, e.g., job applications followed by interviews. The urn-ball matching function arises endogenously under directed search from one-to-many meetings. That is, market organizers direct visitors' search, and each site can meet many visitors before choosing one to form a match (see Herreiner 1999; Julien, Kennes, and King 2000; Burdett, Shi, and Wright 2001). In such an environment, matching failures arise from the lack of coordination among visitors. The difficulty to coordinate is lower and the number of matches is higher if the short side directs search than if the long side directs. In contrast, the telephone matching function arises from one-to-one meetings (see Burdett et al. 1995). If the two sides have the same search efficiency, the number of matches is the same regardless of which side organizes trade. Moreover, increasing the relative efficiency of visitors to organizers makes the matching function favor the long side. With this exposition, Theorem 2 has the following implication:

- **Prediction 4**: If the meeting process has one-to-many meetings, the market should be organized by the short side even if the site cost is zero. If matches are generated by one-to-one meetings and if the organizers have sufficiently lower search efficiency than the visitors, then the market should be organized by the long side. In both cases, the short side is elastic if and only if the elastic side's participation cost is high.

In the labor market, since one-to-many meetings are common, Prediction 4 implies that firms should organize trade and be on the short side. This implication strengthens Predictions 1 and 2, because it holds even when the site cost is zero. In contrast, in the marriage/dating market, one-to-one meetings are common, and so the number of matches can be approximated by the telephone matching function. In this market, trade can be organized by either side, depending on the site cost and the relative elasticity of the two sides. If the site cost and the relative elasticity of the organizers to visitors are both small, then trade should be organized by the long side.

The analysis sheds light on how trading platforms affect the efficient market organization. Platforms are common in online trading, where a third party creates trading sites and charges fees for usage. Network externalities are an important feature of trading platforms, that have been emphasized by Rochet and Tirole (2003). We examine other differences between a platform and a traditional trading site (see Online Appendix F). On the surface, a significant difference of a platform from a traditional trading site is that the platform fees shift the cost from the site cost to the participation cost. However, this feature alone does not change the efficient market organization, because the planner can use transfers between the sides to neutralize the effect of the fees. For an efficient organization, the relevant changes are technological and, particularly, changes to the matching function. For example, in the marriage/dating market, the Internet can change the meeting process from one-to-one meetings to one-to-many meetings, because a posted profile can attract more than one person before one is selected. At the same time, the Internet may increase the search efficiency of a visitor relative to an organizer, since a visitor can visit many sites quickly. These changes affect the matching function in opposite directions. While the change to one-to-many meetings favors the short side, the increase in a visitor's relative search efficiency favors the long side.

A trading platform can also change the cost and the capacity of sites. By reducing the site cost, a platform increases the likelihood that the market will be organized by the inelastic side, as analyzed in Section III.B. By increasing the site capacity per organizer, a platform effectively reduces the inelasticity of sites in the market organized by the inelastic side and, hence, increases the efficiency of such a market.

V. Heterogeneous Individuals and Coexistence of Markets

Markets organized by the two sides coexist for assets in reality. The benchmark model does not generate such coexistence generically. For the coexistence, we introduce heterogeneity on the inelastic side.[18] Suppose that net surplus in a match is u_H for a fraction ϕ of inelastic individuals and u_L for the remaining fraction of inelastic individuals. The type of an inelastic individual is public information. Recall that u is the utility of consumption minus the postmatch production cost. If inelastic individuals are buyers, then a type H individual is a buyer with higher utility; if inelastic individuals are sellers, then a type H individual is a seller whose product has higher quality or whose postmatch production cost is lower. Assume $u_H > u_L > k + \min\{c_n, c_e\}$ so that a trade is beneficial even for a low-value inelastic individual if participation costs are sufficiently low. Normalize the costs by the joint surplus in a match as

$$C_i(u) = \frac{c_i}{u} \text{ and } K(u) = \frac{k}{u}, \text{ where } i \in \{e, n\}.$$

We shorten the notation $C_i(u_j)$ to C_{ij} and $K(u_j)$ to K_j, where $i \in \{e, n\}$ and $j \in \{L, H\}$. Also, let $\phi_H = \phi$ and $\phi_L = 1 - \phi$.

Let market ij denote the market that is organized by side i and has type j inelastic individuals, where $i \in \{e, n\}$ and $j \in \{L, H\}$. In market ij, let θ_{ij} be the ratio of elastic to inelastic individuals, and W_{ij} be social welfare per inelastic individual. Modifying the analysis in Section II.B, we have:

$$\begin{aligned} W_{ej} &\equiv \left[f\left(C_{ej} + K_j\right) - C_{nj} \right] u_j, \\ W_{nj} &\equiv \left[f^{-1}\left(C_{ej}\right) - K_j - C_{nj} \right] u_j. \end{aligned}$$

Market ej dominates market nj if $W_{ej} > W_{nj}$, market nj dominates market ej if $W_{nj} > W_{ej}$, and the markets are welfare equivalent

18. One may introduce heterogeneity on both sides as in Shi (2001), who has examined efficient sorting with search frictions but assumed that firms direct search. Although this extension is exciting, it would take the analysis too far afield and hence is left for future research.

if $W_{ej} = W_{nj}$. The inequality $W_{nj} > W_{ej}$ can be written as $G(C_{ej}, K_j) > 0$, where G is defined in equation (8). As in the benchmark model, the case $W_{ej} = W_{nj}$ occurs only in a measure-zero set of parameters. Thus, it is generically inefficient to have two market organizations for the same type of inelastic individuals. However, the efficient market organization can differ between the two types of inelastic individuals. As before, we assume that for each $j \in \{L, H\}$, $\max\{W_{ej}, W_{nj}\} > 0$ so that at least one market is viable for each j. Equivalently, this assumption is

$$C_{nj} < \max\{f\left(C_{ej} + K_j\right), f^{-1}\left(C_{ej}\right) - K_j\} \text{ for } j \in \{L, H\}. \tag{11}$$

Maintain Assumption 3 so that $G(x, y) = 0$ has a unique generic interior solution of x for any given $y \in (0, 1 - x)$.

To economize on space, we assume $k > k_b$ if the matching function favors the long side and $k > 0$ if the matching function is symmetric. Under Assumption 3, $G(C_{ej}, K_j) > 0$ if and only if $C_{ej} < c_d(K_j)$, where $c_d(K)$ is defined similarly to equation (9) by

$$f\left(c_d + K\right) = c_d. \tag{12}$$

Using the definitions of (C_{ej}, K_j), we express the condition $C_{ej} < c_d(K_j)$ as

$$\frac{c_e}{u_j} < c_d\left(\frac{k}{u_j}\right). \tag{13}$$

Note that $c_d(K)$ defined by equation (12) is a strictly decreasing function, and so $c_d(\frac{k}{u_j})$ strictly increases in u_j for any given k. If equation (13) is satisfied for $j = L$, then it is also satisfied for $j = H$. There are parameter values with which equation (13) is satisfied for $j = L$ and violated for $j = H$. We summarize these results in the following theorem and omit the proof:

THEOREM 3. Maintain Assumptions 1 and 3 and inequality (11). If $c_e < u_L c_d(\frac{k}{u_L})$, inelastic individuals should organize all trade with coexisting markets nL and nH. If $c_e > u_H c_d(\frac{k}{u_H})$, elastic individuals should organize all trade with coexisting markets eL and eH. If $u_L c_d(\frac{k}{u_L}) < c_e < u_H c_d(\frac{k}{u_H})$, market nH organized by type H inelastic individuals and market eL organized by elastic individuals coexist.

The intuitive explanation for these results is similar to those in Sections III and IV. In the case we focus on here, the matching function does not favor the long side strongly, so trade should be organized by the short side. If elastic individuals' participation cost is low, it is socially desirable to have a relatively large measure of elastic individuals enter the market to be on the long side. This is achieved by having inelastic individuals incur the site cost to organize all trade in two submarkets, one by each type of inelastic individuals. If elastic individuals' participation cost is high, the social optimum asks elastic individuals to be on the short side to organize all trade in two submarkets, one for each type of inelastic individuals to visit. When elastic individuals' participation cost is intermediate, high-value inelastic individuals organize market nH while low value inelastic individuals visit market eL organized by the elastic side. By incurring the site cost to organize trade, high-value inelastic individuals are on the short side of market nH so that they have relatively high matching probability to realize the high value. In contrast, low-value inelastic individuals are on the long side of market eL so that elastic individuals in the market trade with relatively high probability. The expected surplus for an elastic individual in market eL is the same as in market nH.

1. **The Asset Market**. The coexistence of the two organizations is common for assets and durables. For many assets, including houses and artwork, sellers are relatively less elastic than buyers. High-end sellers may pay brokers to sell their assets and low-end sellers may wait for buyers to come. For financial assets, a main determinant of a seller's type is the need for liquidity. Asset sellers who have urgent needs for liquidity may pay the cost to actively seek buyers, whereas sellers who do not have immediate liquidity needs may wait for buyers to contact them. We list this implication of Theorem 3 as follows:

- **Prediction 5**: Asset holders with high liquidity needs are likely to organize trade while asset holders with low liquidity needs are likely to await buyers to initiate trade.

An example is the market for repurchase agreements, a short-term loan market where one party sells collateral securities temporarily for money and agrees to buy back the collateral at a preset price and time. The amount of outstanding loans in this market is in trillions of dollars despite the large fall in the 2008–2009 recession. In this market, trading is decentralized. An owner of the securities intends to hold the securities to maturity instead of selling them outright before maturity. However, the owner may have temporary needs to borrow money, for example, to close another deal. Such an owner can sell the securities in the repo market. On the other side, a buyer of the securities may need the securities to temporarily cover a short position or simply to earn interest through lending. A repurchase agreement is called a repo to the seller of collateral securities (the borrower) and a reverse to the buyer of collateral (the lender). A repo is typically initiated by the borrower and a reverse by the lender. The cost of initiating a deal can be treated as a site cost. In this market, the supply is endogenous on both sides, but sellers are relatively less elastic because their needs for short-term liquidity are more pressing. In contrast, a depository institution that has unexpected idle money may or may not enter the repo market to lend. Moreover, borrowers are heterogeneous in the need for liquidity.

Prediction 5 is consistent with the observation on dealers in the repo market for U.S. Treasury securities. These dealers initiate both repos and reverses. Relative to other participants, dealers are more likely to face liquidity shortage in the short term, such as overnight. As a result, dealers consistently borrow more money doing overnight and open repos than they lend doing overnight and open reverses (Stigum and Crescenzi 2007). In term repos and reverses, which have longer maturities than overnight and open repos, the liquidity needs change for a dealer relative to a nondealer. As a result, the trading pattern is reversed, where dealers consistently lend more money doing term reverses than they borrow doing term repos.

Theorem 3 also helps us understand the growth of the repo market induced by innovations and regulatory reforms. A main innovation is the general collateral finance introduced in 1998 by the Fixed Income Clearing Corporation, a clearing agency for U.S. government securities. This system allows borrowers and lenders to settle their daily transactions on a net basis, instead of the gross settlement used previously. It also increases the flexibility for borrowers to substitute the collateral in a contract with similar securities in case they fail to find the specified collateral in time. Partly because of these changes, outstanding loans in the repo market multiplied between 2000 and 2006. For dealers in the market for the U.S. government securities, the amount of borrowing through repos and the amount of lending through reverses grew. The difference between the two—net repo financing by dealers—tripled over these six years (Stigum and Crescenzi 2007). To use Theorem 3 to explain this growth, note that netting reduces the settlement cost and hence increases the match surplus u. In contrast, the flexibility in collateral substitution reduces the site cost to initiate a contract, since a borrower does not necessarily need to have the particular securities in place to initiate a repo. The increase in u and the reduction in k increase the gains from trade and stimulate the growth of the market. As analyzed before, the reduction in k increases the benefits for the inelastic side to be the market organizers relative to the elastic side. With dealers being interpreted as being relatively inelastic (see above), this may explain why net repo financing by dealers increased.

VI. Market Implementation

Can a realistic market mechanism implement the social optimum? If the answer is negative, then we can investigate what corrective policies are needed. If the answer is affirmative, it gives further support for the mapping between our results and reality. In addition, a market mechanism reveals how prices divide the match surplus. We start with the benchmark model where all individuals on each side are homogeneous. Consider a realistic mechanism in which market organizers post prices to compete for customers (e.g., Peters 1991; Montgomery 1991). Each price p is associated with a ratio θ, that is, the measure of elastic individuals per inelastic individual at the particular price. When an organizer chooses p or when a visitor chooses which organizer to visit, the individual takes into account the dependence of θ on p. In this sense, the terms of trade (p, θ) direct visitors' search. To simplify the description, we refer to the group of organizers who post the same p together with the visitors to such organizers as submarket (p, θ). In a submarket, the matching function M determines the measure of matches. In the equilibrium, the ratio θ must be consistent with individuals' optimal choices.

In both market e and market n, an organizer chooses (p, θ) to maximize an inelastic individual's payoff, subject to the constraint that an elastic individual's net expected profit of participating in the market is nonnegative. In market e, this choice solves:[19]

$$\max_{(p,\theta)} F(\theta)(1-p) - c_n \text{ s.t. } \frac{F(\theta)}{\theta} p \geqslant c_e + k. \tag{14}$$

In market n, the choice of an organizer (i.e., an inelastic individual) solves:[20]

$$\max_{(p,\theta)} \theta F\left(\frac{1}{\theta}\right)(1-p) - c_n - k \text{ s.t. } F\left(\frac{1}{\theta}\right) p \geqslant c_e. \tag{15}$$

In both markets, the quintessential feature is that an individual makes the trade-off between the price and the matching probability. Note that the surplus division is endogenous, with the elastic side's share being equal to the price.

In equations (14) and (15), the constraint holds with equality because of free entry of elastic individuals. Solving the price p from such an equality, we can verify that $\theta_e(c_e + k)$ defined in equation (2) is the solution to equation (14) and $\theta_n(c_e)$ defined in equation (6) is the solution to equation (15). Moreover, the maximized expected payoff of an inelastic individual is $W_e = f(c_e + k) - c_n$ in market e and $W_n = f^{-1}(c_e) - c_n - k$ in market n. Each inelastic individual chooses to participate in the market that yields the relatively higher payoff. Because W_n and W_e in the equilibrium are the same as in the social optimum, the market organization in the equilibrium is the same as in the social optimum. Thus, we have proven the following proposition:

PROPOSITION 1. In the economy where each side of the market contains homogeneous individuals, the market equilibrium in which individuals compete to offer (p, θ) to direct search implements the social optimum.

19. When an elastic individual chooses whether to participate in the market, the individual also chooses which submarket to enter. Thus, the participation cost is not sunk at the time of choosing the submarket. This is why c_e appears in the constraint in equation (14).

20. It is immaterial which side's payoff is used as the objective function. In particular, in market n, we can formulate the choice problem as $\max_{(p,\theta)} [F(\frac{1}{\theta})p - c_e]$ s.t. $\theta F(\frac{1}{\theta})(1-p) - c_n - k = \Delta_n$, where Δ_n is an inelastic individual's payoff in the market. This problem is dual to the primal problem equation (15). In the equilibrium, free entry of elastic individuals determines Δ_n by forcing such an individual's payoff to zero. With such Δ_n, the dual problem and the primal problem have the same solution.

Given the market organization, it is well known that directed search can induce efficient entry of elastic individuals by internalizing matching externalities (e.g., Moen 1997; Acemoglu and Shimer 1999; Shi 2001). The equilibrium price divides the match surplus endogenously in a way that compensates each side of the market with the side's share in the matching function—a condition established by Hosios (1990). What is new in Proposition 1 is that directed search also induces the efficient market organization. In both market e and market n, competitive entry of elastic individuals drives their expected gain from participation to zero. As a result, social welfare in each market is equal to an inelastic individual's expected gain from participation. The market that maximizes this expected gain wins the competition because individuals choose which market to participate in.

The implementation of the efficient allocation can be extended to the economy in Section V where there are two types of inelastic individuals. In this extension, the markets are indexed by ij, where $i \in \{e, n\}$ is the side of the organizers and $j \in \{L, H\}$ is inelastic individuals' type. In market ij, the terms of trade are (p_{ij}, θ_{ij}) across submarkets. Elastic individuals enter the market competitively, and each earns zero net expected profit from entry. An analysis similar to the foregoing shows that the equilibrium allocation coincides with the social optimum.

1. **Unequal Welfare Weights**. In the analysis so far, we have used an egalitarian social welfare function. If the planner puts unequal weights on the two sides of the market, are the results on the social optimum robust, and does the equilibrium of directed search still implement the social optimum? The answer is affirmative to the first question, but positive to the second question only under a qualification. The following proposition lists the results (see Online Appendix E for a proof):

PROPOSITION 2. Let $\lambda \in (0, \infty)$ be the welfare weights for the elastic side relative to the inelastic side. The efficient allocation under unequal welfare weights $\lambda \neq 1$ is the same as under equal welfare weights and can be implemented by the above equilibrium with directed search. However, equilibrium welfare is the same as in the social optimum only when $\lambda \leqslant 1$. When $\lambda > 1$, the expected surplus is inefficiently low for elastic individuals and inefficiently high for inelastic individuals.

When the two sides of the market have different welfare weights, the division of the match surplus matters for social welfare. The planner chooses this division, in addition to θ, to maximize social welfare under individual rationality constraints. Because utility is transferable, the planner can increase social welfare by shifting the match surplus from the side with lower welfare weights to the side with higher welfare weights. That is, the planner maximizes social welfare by setting the expected surplus of participating in the market to zero for the side that has lower welfare weights. With this efficient division of the surplus, social welfare depends only on the sum of expected match surpluses. Maximizing this sum, the efficient θ under unequal welfare weights is identical to the one under the egalitarian welfare function. So is the efficient market organization. Moreover, the foregoing equilibrium with directed search induces the same entry of elastic individuals (θ) and the same market organization as in the social optimum.

Although the equilibrium allocation is efficient, equilibrium welfare may differ from the social optimum. Because competitive entry of elastic individuals drives expected surplus to zero for these individuals, equilibrium welfare is the same as in the social optimum only when the elastic side's welfare weights are less than or equal to the inelastic side's. When the elastic side has higher welfare weights than the inelastic side, the equilibrium generates inefficiently low welfare for elastic individuals and for the economy as a whole. To restore efficient welfare, a planner can use a price subsidy to elastic individuals financed by a tax on inelastic individuals, together with permits that restrict entry of elastic individuals. A subsidy to elastic individuals alone is ineffective because it will be competed away.

VII. Conclusion

Return to the question in the title: should buyers or sellers organize trade in a frictional market? The answer depends on the relative elasticity of the supply of buyers to sellers, the participation cost of the elastic side, the cost of a trading site, and the (a)symmetry in the matching function. When the site cost is positive or the matching function favors the short side of the market, the market should be organized by the short side. When the site cost is sufficiently small and the matching function strongly favors the long side, the market should be organized by the long side. In both cases, the elastic side is short if and only if the elastic side's participation cost exceeds a threshold that decreases in the site cost. The efficient organizers can change from the elastic to the inelastic side if the site cost or the elastic side's participation cost falls sufficiently, or if the search efficiency of organizers relative to visitors falls sufficiently. These results provide a unified explanation for why trade has often been organized by sellers in the goods market and by buyers (firms) in the labor market, and how the efficient organization changes with technological advances. Moreover, we introduce heterogeneity on the inelastic side to show that markets organized by the two sides can coexist, as in the asset market. Finally, we formulate a directed-search equilibrium to implement the social optimum.

A useful extension of the article is to introduce private information. For the asset market where sellers have private information about the assets, Guerrieri, Shimer, and Wright (2010) and Chang (2018) assume that uninformed individuals (buyers) post the trading mechanism to direct search. Such a market faces the problem of adverse selection. In contrast, Delacroix and Shi (2013) assume that informed individuals post the trading mechanism to direct search. Such a market faces a signaling problem. In general, the decision to trade may reveal information, for example, Wolinsky (1990). Different market organizations may differ in the ability to separate heterogeneous individuals and mitigate information frictions. The current article shows that different market organizations also differ in the ability to mitigate matching frictions and trading costs. It is interesting to examine how these two roles of a market organization interact. Specifically, the two sides can differ in both how elastic and how informed they are. It is an open question which combination of these two characteristics makes a side the efficient market organizers.

Appendix A: Properties of (h, f), the Matching Function, and Assumption 3

A.1. Properties of h and f Defined in Equation (4)

Lemma 1. Maintain Assumption 1 and recall $A = F'(0) \leqslant 1$.

(i) $h(\theta)$ defined in equation (4) satisfies $h'(\theta) = -\theta F''(\theta) > 0$, $h(0) = 0$, and $h(\infty) = 1$.

(ii) $f(x)$ defined in equation (4) satisfies $f(0) = 1$, $f(A) = 0$, $f'(x) = -\theta_e(x) < 0$, and $f''(x) = -\frac{1}{F''(\theta_e(x))} > 0$ for all $x \in [0, A)$.

For $x > A$, $f'(x) = f''(x) = 0$.
(iii) $f(x) \leqslant 1 - x$ for all $x \in [0, 1]$, where the inequality is strict for $x \in (0, 1)$. Moreover, $\max\{f(c + k), f^{-1}(c) - k\} \leqslant 1 - k - c$ for all $c \in [0, 1 - k]$, where the inequality is strict for all $c \in (0, 1 - k)$.
(iv) If $A < 1$, there exists a unique $x_0 \in (0, A)$ such that

$$A - x_0 - f(x_0) = 0 \text{ and } f'(x_0) < -1. \tag{16}$$

$A - x - f(x) < 0$ for $x \in [0, x_0) \cup (A, 1]$, and $A - x - f(x) > 0$ for $x \in (x_0, A)$. If $A = 1$, set $x_0 = 0$ so that $A - x - f(x) > 0$ still holds for all $x \in (x_0, A)$.
(v) h and f satisfy

$$f\left(F'(\theta)\right) = h(\theta) \quad \text{for all } \theta \in [0, \infty), \tag{17}$$

$$f^{-1}(x) = F'(h^{-1}(x)) \quad \text{for all } x \geqslant 0. \tag{18}$$

Proof. Under Assumption 1, the properties of h and f in (i) and (ii) of Lemma 1 can be verified from equations (2) and (4). For (iii), temporarily denote $Q(x) = 1 - x - f(x)$. Part (ii) implies $Q''(x) < 0$ for all $x \in (0, A)$ and $Q(x) = 1 - x$ for $x \in (A, 1]$. Also, $Q(0) = Q(1) = 0$. Thus, $Q(x) \geqslant 0$ for all $x \in [0, 1]$, where the inequality is strict for all $x \in (0, 1)$. Setting $x = c + k$ and $x = 1 - c$ in turn, this result yields $f(c + k) \leqslant 1 - k - c$ and $f^{-1}(c) - k \leqslant 1 - k - c$ for all $c \in [0, 1 - k]$, where the inequalities are strict if $c \in (0, 1 - k)$. Therefore, $\max\{f(c + k), f^{-1}(c) - k\} \leqslant 1 - k - c$ for all $c \in [0, 1 - k]$, where the inequality is strict for all $c \in (0, 1 - k)$.

For (iv), suppose $A < 1$ and temporarily denote $Q(x) = A - x - f(x)$. Then, $Q''(x) < 0$ for $x \in (0, A)$ and $Q(x) = A - x$ for all $x \in [A, 1]$. Since $Q(0) = A - 1 < 0$, $Q(A) = 0$ and $Q'(A) < 0$, there is a unique $x_0 \in (0, A)$ as defined in equation (16) such that $Q(x_0) = 0$ and $Q'(x_0) > 0$. Then, $A - x - f(x) < 0$ for $x \in [0, x_0) \cup (A, 1]$, and $A - x - f(x) > 0$ for $x \in (x_0, A)$. As A increases toward 1, x_0 decreases toward 0 and $f'(x_0)$ increases toward -1, but $A - x - f(x) > 0$ still holds for all $x > x_0$ in the limit $A \nearrow 1$.

For (v), the definition of $\theta_e(x)$ in equation (2) implies $\theta_e(F'(\theta)) = \theta$ for all $\theta \in [0, \infty)$. The definition of f implies $f(F'(\theta)) = h(\theta_e(F'(\theta))) = h(\theta)$, as stated in equation (17). Setting $\theta = h^{-1}(x)$, we have $f(F'(h^{-1}(x))) = h(h^{-1}(x)) = x$ for all $x \geqslant 0$. Thus, $f^{-1}(x) = F'(h^{-1}(x))$ for all $x \geqslant 0$, as stated in equation (18). □

A.2. Properties of the Matching Function

LEMMA 2. Define $k_0 \equiv 2F'(1) - F(1)$.

(i) If the matching function is symmetric, then $A = 1$ and $k_0 = 0$, where $A = F'(0)$. In addition, $\theta_e(x) = \theta_n(x)$ and $f^{-1}(x) = f(x)$ for all $x \geqslant 0$. If the matching function favors the short side, then $A = 1$ and $k_0 \leqslant 0$. If the matching function favors the long side, then $A \leqslant 1$ and $k_0 \geqslant 0$.

(ii) With $k \in (0, 1)$, the condition $f(k) < A - k$ is satisfied if the matching function is symmetric, or favors the short side, or favors the long side with $k > x_0$, where $x_0 > 0$ is defined in (iv) in Lemma 1. The condition $f(k) > A - k$ is satisfied when the matching function favors the long side with $k < x_0$.

(iii) For all matching functions, $G'_c(c, k) = \theta_e(c + k) - \theta_n(c)$ for all c, and $\theta_e(c_d + k) = \frac{1}{\theta_n(c_d)}$, where G is defined in equation (8) and c_d in (9).

(iv) The urn-ball matching function in Example 1 has $A = 1$ and $k_0 = 1 - 3e^{-1} < 0$, and the telephone matching function has $k_0 = (A^{-1} - 1)(A^{-1} + 1)^{-2}$.

Proof. Define $D(\theta) \equiv F(\theta) - \theta F\left(\frac{1}{\theta}\right)$. Then, a matching function is symmetric if $D(\theta) = 0$ for all $\theta \geqslant 0$, favors the short side of the market if $D(\theta) > 0$ is equivalent to $\theta \in (0, 1)$, and favors the long side if $D(\theta) > 0$ is equivalent to $\theta \in (1, \infty)$.

(i) We first prove $\theta_e(x) = \theta_n(x)$ and $f^{-1}(x) = f(x)$ for all $x \geqslant 0$ if the matching function is symmetric. When the matching function is symmetric, $D(\theta) = 0$ for all θ and hence $D'(\theta) = 0$ for all θ. Writing $D'(\theta) = 0$ as $F'(\theta) = h\left(\frac{1}{\theta}\right)$ and setting $\theta = \theta_e(x)$, we have $h(\frac{1}{\theta_e(x)}) = F'(\theta_e(x)) = x$, where the second equality comes from equation (2). Thus, $h^{-1}(x) = \frac{1}{\theta_e(x)}$. Since $h^{-1}(x) = \frac{1}{\theta_n(x)}$ by equation (6), then $\theta_e(x) = \theta_n(x)$. In addition,

$$f^{-1}(x) = F'\left(h^{-1}(x)\right) = F'\left(\frac{1}{\theta_e(x)}\right) = h(\theta_e(x)) = f(x) \text{ for all } x \geqslant 0.$$

The first equality is equation (18), the second equality was just derived above, the third equality uses $D'(\frac{1}{\theta_e}) = 0$, and the last equality is the definition of f.

Now consider any matching function. To prove the results on (A, k_0), differentiate $D(\theta)$ to obtain $D'(\theta) = F'(\theta) - h\left(\frac{1}{\theta}\right)$, where h is defined in equation (4). With the properties in Lemma 1, we can verify that $D(0) = D(1) = 0$, $D'(0) = A - 1$, and $D'(1) = k_0$. If the matching function is symmetric, then $D(\theta) = 0$ for all θ, which implies $D'(\theta) = 0$ for all θ. In particular, $D'(0) = D'(1) = 0$; that is, $A = 1$ and $k_0 = 0$. If the matching function favors the short side, then $D(\varepsilon) > 0$ and $D(1 - \varepsilon) > 0$, where $\varepsilon > 0$ is arbitrarily small. Since $D(0) = 0$, then $\frac{1}{\varepsilon}\left[D(\varepsilon) - D(0)\right] > 0$. Taking the limit $\varepsilon\downarrow 0$ yields $D'(0) \geqslant 0$, that is, $A \geqslant 1$. Because $A \leqslant 1$ by Assumption 1, then $A = 1$. Similarly, $D'(1) \leqslant 0$, so $k_0 \leqslant 0$. If the matching function favors the long side, then $D'(0) \leqslant 0$ and $D'(1) \geqslant 0$, which lead to $A \leqslant 1$ and $k_0 \geqslant 0$.

(ii) Recall that $f(k) < 1 - k$ for all $k \in (0, 1)$ (see (iii) of Lemma 1). Take any $k \in (0, 1)$ as given. If the matching function is symmetric or favors the short side, then $A = 1$, so $f(k) < 1 - k = A - k$. The inequality $f(k) < A - k$ also holds if the matching function favors the long side with $k > x_0$, where x_0 is defined in (iv) in Lemma 1. In contrast, the inequality $f(k) > A - k$ holds only when the matching function favors the long side with $k < x_0$.

(iii) For all matching functions and all c, calculate

$$f'(c+k) = -\theta_e(c+k), \quad [f^{-1}(c)]' = -\theta_n(c). \tag{19}$$

Thus, $G'_c(c,k) = \theta_e(c+k) - \theta_n(c)$. Because c_d is defined by $f(c_d + k) = c_d$, which is equivalent to $F'^{-1}(c_d + k) = h^{-1}(c_d)$, then $\theta_e(c_d + k) = \frac{1}{\theta_n(c_d)}$.

(iv) The expressions for k_0 and A for the urn-ball and the telephone matching functions in Example 1 can be verified directly. □

A.3. *A Lemma Related to Assumption 3*

Examine the function $G(c, k)$ defined in equation (8). A solution c to $G(c, k) = 0$ is generic if $G'_c(c,k) \neq 0$, and interior if $c \in (0, \bar{c})$, where

$$\bar{c} \equiv \max\{A - k,\ f(k)\}. \tag{20}$$

LEMMA 3. (i) $G'_c(c_d, k)$ has the same sign as $(k_0 - k)$, where c_d is defined in (9) and k_0 in Lemma 2.

(ii) Under Assumption 3, c_d is the unique generic interior solution of c to $G(c, k) = 0$ under all symmetric matching functions for $k > 0$ and under all asymmetric matching functions for $k \geqslant 0$.

(iii) If the matching function is symmetric, Assumption 3 is satisfied for all $k \geqslant 0$.

(iv) If the matching function is asymmetric, a necessary condition for Assumption 3 is $(k - k_0)[A - k - f(k)] \geqslant 0$.
(v) For all $k \geqslant 0$, Assumption 3 is satisfied under the urn-ball matching function in Example 1.

Proof. (i) By (iii) in Lemma 2, $G_c'(c_d, k) = \theta_e(c_d + k) - \frac{1}{\theta_e(c_d+k)}$. Thus, $G_c'(c_d, k) > 0$ if and only if $\theta_e(c_d + k) > 1$. By the definition of θ_e in equation (2), this condition is equivalent to $c_d < F'(1) - k$ which, by the definition of c_d, is equivalent to $f(F'(1)) < F'(1) - k$. By equation (17), $f(F'(1)) = h(1)$. Thus, $G_c'(c_d, k) > 0$ if and only if $k < F'(1) - h(1) = k_0$ (see Lemma 2). That is, $G_c'(c_d, k)$ has the same sign as $(k_0 - k)$. Thus, Assumption 3 is equivalent to the requirements that $G_c'(c, k)$ should have the same sign as $(k_0 - k)$ at all interior solutions of c to $G(c, k) = 0$ and, if the matching function is asymmetric, then $k \neq k_0$ for all $k \geqslant 0$.

(ii) Consider all symmetric matching functions for $k > 0$ and all asymmetric matching functions for $k \geqslant 0$. If the matching function is symmetric, then $k_0 = 0$ and $k_0 - k < 0$. If the matching function is asymmetric, then the second part of Assumption 3 requires $k_0 - k \neq 0$. In all cases under consideration, $G_c'(c_d, k) \neq 0$, and so c_d is a generic interior solution of c to $G(c, k) = 0$. Since the first part of Assumption 3 requires $G_c'(c, k)$ to have the same sign as $(k_0 - k)$ at all interior solutions of c to $G(c, k) = 0$, then the interior solution is unique: if there are multiple interior solutions, the sign of $G_c'(c, k)$ must switch signs between two adjacent solutions. Because c_d is an interior solution to $G(c, k) = 0$, then c_d is the unique solution under Assumption 3.

(iii) Consider any symmetric matching function and all interior c. The symmetry in the matching function yields $A = 1$, $k_0 = 0$, and $f^{-1}(x) = f(x)$ for all $x \geqslant 0$ (see Lemma 2). Thus, $\bar{c} = f(k) \leqslant 1 - k$ and $k_0 - k = -k$. Moreover, $G(c, k) = f(c) - k - f(c + k)$. Compute $G_{ck}''(c, k) = -f''(c + k) < 0$, where the strict inequality comes from $c < \bar{c}$ and (ii) in Lemma 1. Thus, $G_c'(c, k) \leqslant G_c'(c, 0) = 0$, with strict inequality if $k > 0$. If $k > 0$, then $G_c'(c, k)$ and $(k_0 - k)$ are both strictly negative. If $k = 0$, then $G_c'(c, 0)$ and $(k_0 - k)$ are equal to 0 and again have the same sign. Since $G_c'(c, k)$ and $(k_0 - k)$ have the same sign for all $k \geqslant 0$ and all interior c, clearly they have the same sign at all solutions of c to $G(c, k) = 0$. Thus, the first part of Assumption 3 is satisfied. The second part is irrelevant when the matching function is symmetric.

(iv) If $k = k_0$, then the condition $(k - k_0)[f(k) + k - A] \geqslant 0$ is clearly satisfied. Consider any asymmetric matching function with $k \neq k_0$. If $k > k_0$, then $G_c'(c_d, k) < 0$. Since $G(c_d, k) = 0$, in this case Assumption 3 requires that $G(c, k) > 0$ for all $c \in (0, c_d)$ and $G(c, k) < 0$ for all $(c_d, \bar{c})$; otherwise, c_d would not be the only interior solution to $G(c, k) = 0$. In particular, $G(\varepsilon, k) > 0$ where $\varepsilon > 0$ is sufficiently small. Taking the limit $\varepsilon \to 0$ yields $G(0, k) \geqslant 0$. Similarly, it is necessary to have $G(\bar{c} - \varepsilon, k) < 0$ where $\varepsilon > 0$ is sufficiently small, which leads to the condition $G(\bar{c}, k) \leqslant 0$. Compute $G(0, k) = A - k - f(k)$ and $G(\bar{c}, k) = f^{-1}(A - k) - k$. Because f is a decreasing function, the two conditions $G(0, k) \geqslant 0$ and $G(\bar{c}, k) \leqslant 0$ are both equivalent to $f(k) \leqslant A - k$. Thus, $(k - k_0)[A - k - f(k)] \geqslant 0$ is necessary for Assumption 3. If $k < k_0$, then $G_c'(c_d, k) > 0$. In this case, Assumption 3 requires $G(0, k) \leqslant 0$ and $G(\bar{c}, k) \geqslant 0$, which can be written as $f(k) \geqslant A - k$. Again, $(k - k_0)[A - k - f(k)] \geqslant 0$ is necessary for Assumption 3.

(v) The urn-ball matching function in Example 1 yields:

$$
\begin{aligned}
&1 - \left(1 + \frac{1}{\theta_e(x)}\right) e^{-\frac{1}{\theta_e(x)}} = x, \quad \theta_n(x) = -\ln x \\
&f(x) = e^{-\frac{1}{\theta_e(x)}}, \quad f^{-1}(x) = 1 - (1 - \ln x)x, \\
&k_0 = 1 - 3e^{-1} < 0, \quad D(\theta) = \theta\left(1 - e^{-\frac{1}{\theta}}\right) - 1 + e^{-\frac{1}{\theta}},
\end{aligned}
$$

where $D(\theta) = F(\theta) - \theta F\left(\frac{1}{\theta}\right)$. Moreover, $A = 1$. With f and f^{-1} above, we have:

$$
G(c, k) = 1 - k - (1 - \ln c)c - e^{-\frac{1}{\theta_e(c+k)}}.
$$

Also, c_d solves $1 - k - (2 - \ln c_d)c_d = 0$. Temporarily denote $z = \frac{1}{\theta_e(c+k)}$ and use the definition of $\theta_e(c + k)$ to substitute $c = 1 - k - (1 + z)e^{-z}$. We rewrite the equation $G(c, k) = 0$ as $\Delta(z) = 0$ where

$$
\Delta(z) \equiv \frac{z}{(1-k)e^z - (1+z)} + \ln\left[1 - k - (1+z)e^{-z}\right].
$$

For any given $k \in (0, 1)$, $c > 0$ if and only if $z > z_0$ where $z_0 > 0$ is the unique solution to $(1 + z_0)e^{-z_0} = 1 - k$. Suppose $z > z_0$. It can be verified that $\Delta\left(z_0^+\right) = \infty$ and $\Delta(\infty) < 0$. Moreover, $\Delta'(z) > 0$ if and only if $z > z_1$ where $z_1 > z_0$ is the unique solution to $(1 - k)e^{z_1} = 1 + z_1(1 + z_1)$. Thus, for any given $k \in (0, 1)$, there is a unique $z_d \in (z_0, z_1)$ that solves $\Delta(z_d) = 0$. This unique solution is $z_d = \frac{1}{\theta_e(c_d+k)}$. If $k \to 0$, then $z_0 \to 0$, $z_1 \to 0$ and $z_d \to 0$. Therefore, for all $k \geqslant 0$, $G(c, k) = 0$ has a unique solution of c under the urn-ball matching function. This solution is c_d. It can be verified that $G'_c(c_d, k) < 0$. Because $k_0 = 1 - 3e^{-2} < 0$, then $k_0 - k < 0$ for all $k \in [0, 1]$. Thus, for all $k \geqslant 0$, the urn-ball matching function satisfies Assumption 3. $\square$

Appendix B. Proofs of Theorems 1 and 2

Proof of Theorem 1. Assume that the matching function is symmetric, as in the theorem. The symmetry in the matching function implies $A = 1$, $k_0 = 0$, and $f^{-1}(x) = f(x)$ for all $x \geqslant 0$ (see Lemma 2). Hence, $G(c, 0) = 0$ for all c. If $k = 0$, then the two organizations yield the same welfare for all (c_e, c_n). Consider the case $k > 0$. In this case, $k_0 - k < 0$, and Assumption 3 implies that c_d is the unique generic interior solution of c to $G(c, k) = 0$ (see Lemma 3). Because $G'_c(c_d, k) < 0$ and $G(c_d, k) = 0$, then $G(c, k) > 0$ for $c \in (0, c_d)$ and $G(c, k) < 0$ for $c \in (c_d, \bar{c})$. That is, the efficient allocation requires the inelastic side to organize trade if $c_e < c_d$ and the elastic side to organize trade if $c_e > c_d$.

To see whether the organizers should be on the short or the long side, recall that the site-visitor ratio is θ_e when the elastic side organizes trade and $\frac{1}{\theta_n}$ when the inelastic side organizes trade. Also, (iii) in Lemma 2 implies $\theta_e(c_d + k) = \frac{1}{\theta_n(c_d)}$ and $G'_c(c_d, k) = \theta_e(c_d + k) - \theta_n(c_d)$. Moreover, $\theta'_e(c + k) < 0$ by equation (2) and $\theta'_n(c) < 0$ by equation (6). Since $G'_c(c_d, k) < 0$, then $\frac{1}{\theta_n(c_d)} = \theta_e(c_d + k) < \theta_n(c_d)$. This implies $\theta_n(c_d) > 1$ and hence $\theta_e(c_d + k) < 1$. If $c_e \in (0, c_d)$, inelastic individuals should organize trade, in which case the site-visitor ratio is $\frac{1}{\theta_n(c_e)} < \frac{1}{\theta_n(c_d)} < 1$. If $c_e \in (c_d, \bar{c})$, elastic individuals should organize trade, in which case the site-visitor ratio is $\theta_e(c_e + k) < \theta_e(c_d + k) < 1$. Thus, for all $c_e \in (0, \bar{c})$ with $c_e \neq c_d$, the organizers should be on the short side.

Finally, since f is a decreasing function, it is evident from equation (9) that c_d is a decreasing function of k. $\square$

Proof of Theorem 2. Assume that the matching function is asymmetric, as in the theorem. Then, $k \neq k_0$ as required by Assumption 3.

(i) Suppose that the matching function favors the short side. Then, $A = 1$, $k_0 \leqslant 0$ and $f(k) \leqslant A - k$ (see Lemma 2). The necessary condition for Assumption 3, $(k - k_0)[A - k - f(k)] \geqslant 0$, is satisfied. Since $k \neq k_0$, then either $k > 0$ or $k = 0 > k_0$. In both cases, $k_0 - k < 0$, $G'_c(c_d, k) < 0$, and c_d is the unique interior solution of c to $G(c, k) = 0$ (see Lemma 3). These features imply that $G(c, k) > 0$ for $c \in (0, c_d)$ and $G(c, k) < 0$ for $c \in (c_d, \bar{c})$. That is, trade should be organized by the elastic side if $c_e > c_d$ and by the inelastic side if $c_e < c_d$. Since $G_c'(c_d, k) < 0$, the same proof as that for Theorem 1 shows that for all $c_e \in (0, \bar{c})$, the organizers should be on the short side if the matching function favors the short side.

(ii) Now suppose that the matching function favors the long side. Then, $A \leqslant 1$ and $k_0 \geqslant 0$. Consider first the necessary condition for Assumption 3, $(k - k_0)[A - k - f(k)] \geqslant 0$. If $k > k_0$, then it is necessary to have $f(k) \leqslant A - k$ which, in turn, requires $k > x_0$ (see Lemma 2). If $k < k_0$, then it is necessary to have $f(k) \geqslant A - k$ which, in turn, requires $k < x_0$ (see Lemma 2). Thus, to satisfy Assumption 3, it is necessary to restrict attention to $k < k_a$ or $k > k_b$, where $k_a \equiv \min\{k_0, x_0\}$ and $k_b \equiv \max\{k_0, x_0\}$.

If $k > k_b$, then $k > k_0$, and so $G_c'(c_d, k) < 0$. The analysis and the result in this case are the same as the above, where the matching function favors the short side. That is, trade should be organized by the elastic side if $c_e > c_d$ and by the inelastic side if $c_e < c_d$. Also, the organizers should be on the short side regardless of whether $c_e > c_d$.

If $k < k_a$, then $k < k_0$, and so $G_c'(c_d, k) > 0$. In this case, $G(c, k) < 0$ for $c \in (0, c_d)$ and $G(c, k) > 0$ for $c \in (c_d, \bar{c})$. That is, trade should be organized by the elastic side if $c_e < c_d$ and by the inelastic side if $c_e > c_d$. The expressions for $\theta_e(c_d + k)$, $\theta_n(c_d)$ and $G_c'(c_d, k)$ in the proof of Theorem 1 are still valid here. However, because $G_c'(c_d, k) > 0$ in the current case, then $\frac{1}{\theta_n(c_d)} = \theta_e(c_d + k) > 1$. If $c_e \in (0, c_d)$, elastic individuals are the organizers, and the site-visitor ratio is $\theta_e(c_e + k) > \theta_e(c_d + k) > 1$. If $c_e \in (c_d, \bar{c})$, inelastic individuals are the organizers, and the site-visitor ratio is $\frac{1}{\theta_n(c_e)} > \frac{1}{\theta_n(c_d)} > 1$. Thus, for all $c_e \in (0, \bar{c})$ and $c_e \neq c_d$, the organizers should be on the long side. □

Appendix C. Generalization of Theorems 1 and 2

We generalize Theorems 1 and 2 to include the case where Assumption 3 is violated. Because Assumption 3 is satisfied when the matching function is symmetric (see Lemma 3), the violation of Assumption 3 necessarily requires the matching function to be asymmetric. We focus on $k \neq k_0$. Define $\Psi : [0, \bar{c}] \to [0, \bar{c}]$ by $\Psi(c) = f(c + k)$ for all $c \in [0, \bar{c}]$, where $\bar{c}$ is defined in equation (20). The following theorem is proven in Online Appendix D:

Theorem 4. Maintain Assumptions 1 and 2, but not Assumption 3. Assume $k \neq k_0$. The number of generic interior solutions of

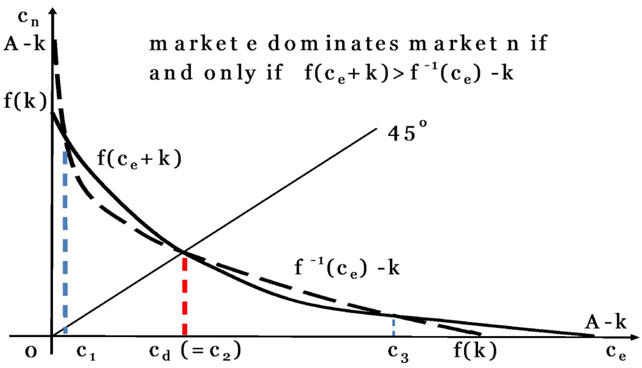

FIGURE A1

Three Crossings between $f^{-1}(c_e) - k$ and $f(c_e + k)$

c to $G(c, k) = 0$ is odd. Denote these generic interior solutions as $\{c_{J+j}\}_{j=-(J-1)}^{J-1}$ with $c_1 < c_2 < ... < c_{2J-1}$, where $J \geqslant 1$ is an integer. Then $c_J = c_d$, and $\Psi(c_{J+j}) = c_{J-j}$ for all j. Let $c_0 = 0$ and $c_{2J} = \bar{c}$. Denote

$$\Omega_1 = \cup\left((c_{J+2\ell-1}, c_{J+2\ell}),\, \ell = -\left\lfloor \tfrac{J-1}{2} \right\rfloor, ..., \left\lfloor \tfrac{J-1}{2} \right\rfloor + 1\right), \tag{21}$$
$$\Omega_2 = \cup\left((c_{J+2\ell}, c_{J+2\ell+1}),\, \ell = -\left\lfloor \tfrac{J}{2} \right\rfloor, ..., \left\lfloor \tfrac{J}{2} \right\rfloor - 1\right),$$

where $\lfloor y \rfloor$ is the largest integer that does not exceed y. If $k = 0$ and the matching function is symmetric, welfare is independent of which side organizes trade. (i) If $k > k_0$, the market should be organized by the inelastic side when $c_e \in \Omega_1$ and by the elastic side when $c_e \in \Omega_2$. (ii) If $k < k_0$, the market should be organized by the elastic side when $c_e \in \Omega_1$ and by the inelastic side when $c_e \in \Omega_2$. In (i) and (ii), the efficient market organizers are short if and only if $G(c_e, k)(c_e - c_d) < 0$, and long if and only if $G(c_e, k)(c_e - c_d) > 0$.

Case (i) in Theorem 4 has $G'_c(c_d, k) < 0$ and thus extends the case depicted in Figure I. Case (ii) has $G'_c(c_d, k) > 0$ and, hence, extends the case depicted in Figure III. To illustrate Theorem 4, let us consider the telephone matching function in Example 1, with $A = 0.65$ and $k = 0.05$. Figure A1 depicts the example. There are three crossings between the two curves, that is, $J = 2$. Because this example has $k_0 = (A^{-1} - 1)(A^{-1} + 1)^{-2} = 0.084 > k$, it belongs to case (ii) of Theorem 4. In particular, the curve $f^{-1}(c_e) - k$ crosses the curve $f(c_e + k)$ from below as c_e increases from slightly below c_d

to slightly above c_d. That is, $G_c'(c_d, k) > 0$. Then, $\Omega_1 = (c_1, c_d) \cup (c_3, A - k)$ and $\Omega_2 = (0, c_1) \cup (c_d, c_3)$. The efficient market organizers are on the elastic side if $c_e \in \Omega_1$, and on the inelastic side if $c_e \in \Omega_2$. The efficient organizers are on the short side for $c_e \in (0, c_1) \cup (c_3, A - k)$ and on the long side for $c_e \in (c_1, c_3)$.

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Supplementary Material

An Online Appendix for this article can be found at *The Quarterly Journal of Economics* online.

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