Solutionnaire

Final ECO 9015

Aut 2019

Question I :

1a) Etats : CO (w/i), a, w

Contrôles : CO' (w'/i'), a', c.

$$\begin{cases} V_w(a,w) = \max\limits_{c,a'} \; u(c) + \beta E_w \Omega_w(a',w') \\ \qquad \text{s.q.} \quad c + a' = w + (1+r)a \\ V_i(a,w) = \max\limits_{c,a'} \; u(c) + \beta E_w \Omega_i(a',w') \\ \qquad \text{s.q.} \quad c + a' = b + (1+r)a \end{cases}$$

$$\begin{cases} \Omega_w(a,w) = \max\{V_w(a,w); V_i(a,w)\} \\ \Omega_i(a,w) = \max\{V_w(a,w); V_i(a,w)\} \end{cases}$$

1b) États : $CO(w/i)$, a, w, z

Contrôles : $CO'(w'/i')$, a', c

$$\begin{cases} V_w(a,w,z) = \max\limits_{c,a'} \; u(c) + \beta \, \mathbb{E}_z \, \Omega_w(a', w', z') \\ \qquad \text{t.q. } c + a' = w + (1+r)a \\ V_i(a,w,z) = \max\limits_{c,a'} \; u(c) + \beta \, \mathbb{E}_z \, \Omega_i(a', w', z') \\ \qquad \text{t.q. } c + a' = z + (1+r)a \end{cases}$$

$$\begin{cases} \Omega_w(a,w,z) = \max\{V_w(a,w,z) \,;\, V_i(a,w,z)\} \\ \Omega_i(a,w,z) = \max\{V_w(a,w,z) \,;\, V_i(a,w,z)\} \end{cases}$$

2a) États : $CO(w/i)$, a, w, z

Contrôles : $CO'(w'/i')$, a', c

$$\begin{cases} V_w(a,w,z) = \max\limits_{c,a'} \; u(c) + \beta \, \mathbb{E}_{z,w} \, \Omega_w(a', w', z') \\ \qquad \text{t.q. } c + a' = w + (1+r)a \\ V_i(a,w,z) = \max\limits_{c,a'} \; u(c) + \beta \, \mathbb{E}_{z,w} \, \Omega_i(a', w', z') \\ \qquad \text{t.q. } c + a' = z + (1+r)a \end{cases}$$

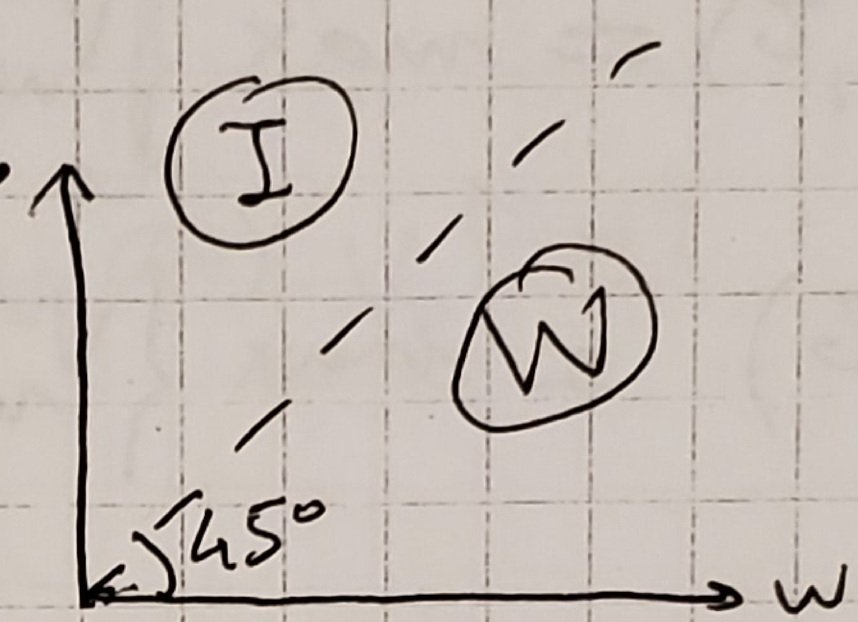
$\Omega_w(a,w,z) = \max\{V_w(a,w,z), V_i(a,w,z)\}$ // $\Omega_i(a,w,z) = \max\left\{\begin{matrix} V_w(a,w,z) - \psi, \\ V_i(a,w,z) \end{matrix}\right\}$

• Quand $\Psi > 0$,

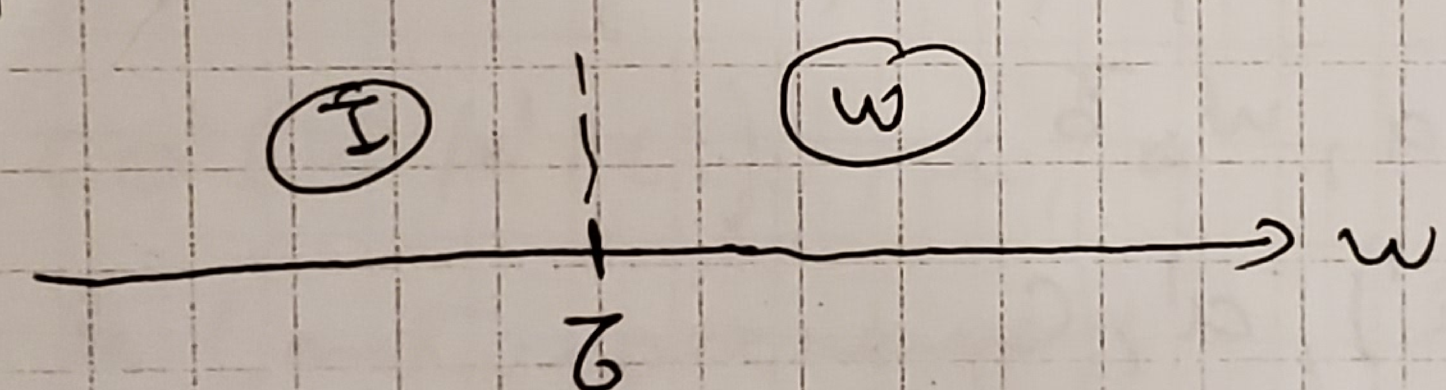
$\Omega_w = \Omega_i$ et le CO est basé sur la comparaison des deux contraintes budgétaires.

$\Rightarrow W \gg I \iff w > z$

Ainsi, pour a donné :



Si, en plus le revenu z est constant :



2.b)

$\begin{cases} \text{Etats de } W : CO = w, a, w, z \\ \text{Etats de } I : CO = i, a, z \end{cases}$

Contrôles : $CO'(w'/i')$, a', c

$$\begin{cases} V_w(a,w,z) = \max\limits_{c,a'} u(c) + \beta \, \mathbb{E}_{w,z} \Omega_w(a',w',z') \\ \qquad \text{t.q. } c + a' = w + (1+r)a \\ V_i(a,z) = \max\limits_{c,a'} u(c) + \beta \, \mathbb{E}_z \Omega_i(a',z') \\ \qquad \text{t.q. } c + a' = z + (1+r)a \end{cases}$$

$$\begin{cases} \Omega_w(a,w,z) = \max\{V_w(a,w,z); V_i(a,z)\} \\ \Omega_i(a,z) = \max\{V_w(a,w,z); V_i(a,z)\} \end{cases}$$

3) Etats : $co(w/i)$, a, w, z

Contrôles : $co'(w'/i')$, a', c

$$\begin{cases} V_w(a,w,z) = \max\limits_{c,a'} u(c) + \beta \, \mathbb{E} \, \Omega_w(a',w',z') \\ \qquad \text{t.q. } c + a' = w + (1+r)a \\ V_i(a,w,z) = \max\limits_{c,a'} u(c) + \beta \, \mathbb{E} \, \Omega_i(a',w',z') \\ \qquad \text{t.q. } c + a' = z + (1+r)a \end{cases}$$

$$\begin{cases} \Omega_w(a,w,z) = \max\{V_w(a,w,z); V_i(a,w,z)\} \\ \Omega_i(a,w,z) = \max\{V_w(a,w,z); V_i(a,w,z)\} \end{cases}$$